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Dynamics of Nonlinear Vibrations in Axially Moving Beams with Rotating Joints

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Abstract—The motion of an axially moving beam with rotating prismatic joint with a tip mass on the end is analyzed to investigate the nonlinear vibration and dynamic stability of the beam. The beam is moving with a harmonic axially and rotating velocity about a constant mean velocity. A time-dependent partial differential equation and boundary conditions with the aid of the Hamilton principle are derived to describe the beam lateral deflection. After the partial differential equation is discretized by the Galerkin method, the method of multiple scales is applied to obtain analytical solutions. Frequency response curves are plotted for the super harmonic resonances of the first and the second modes. The effects of nonlinear term and mean velocity are investigated on the steady state response of the axially moving beam. The results are validated with numerical simulations.

I. Introduction

AXIALLY moving beam with rotating prismatic-joint models may be used for many engineering devices, e.g. robots applications, telescopic members of loading vehicles, space craft antenna, magnetic tape drivers, printers, flexible transmission lines, band saws, weaving mechanisms and furnace conveyor belts all are classified as axially moving beams with rotating prismatic joint.

There are many researches which have been carried out on axially moving systems in literatures. Yuh and Young [1] considered a rotational and translational motion beam. They derived a time-dependent partial differential equation and the boundary conditions for describing the lateral deflection of the beam. They also derived approximated model for multivariable control by using the assumed mode method. They performed validation study for the approximated model by the experiment. Dynamic response of the elastic beam undergoing various motions was investigated by computer simulation. Tadikonda and Baruh [2] considered a complete dynamic model of a translating elastic beam, with a tip mass at one end. They assumed the elastic arm reciprocate in a rigid prismatic joint. Al-Bedoor and Khulief [3] used a finite element dynamic model for a translating and rotating of an elastic beam. They considered all the inertia coupling terms for the model and also considered the time-dependent boundary conditions by the prismatic joint constraints. They compared their numerical simulations with results of other methods for demonstration of the validity and accuracy of their developed model. Kalyoncu [4] investigated a mathematical modeling and dynamic response of a flexible robot manipulator with rotating-prismatic joint and a sliding flexible arm with a tip mass. He assumed the flexible arm to be an Euler-Bernoulli beam with an end-mass. He developed a computer program for solving the numerical simulations. Khadem and Pirmohammadi [5] used a mathematical three-dimensional (3D) model having both revolute and prismatic joints. They studied longitudinal, transversal, and torsional vibration characteristics of the elastic beam. In order to obtain an analytical solution of the vibrational equations, they used the perturbation method. By solving the equations of motion, they showed that mode shapes of the beam with prismatic joints can be modeled as the equivalent clamped beam at each time instant. Chung et al. [6] investigated the dynamic stability of the flapwise motion with rotary oscillation. They studied the linear partial differential equation of flapwise motion to consider the stiffening effect due to the centrifugal force. They used the Galerkin method to discrete the partial differential equation, and the method of multiple scales is applied. By using this method, numerical examples are presented to show the stability of the beam with variations of the oscillating frequency and the maximum angular speed. Wang and Wei [7] studied the vibration of a moving slender prismatic beam. They used Galerkin approximation method with timedependent basis functions for solving the equation of motion. They found that the extending and contracting motions have destabilizing and stabilizing effects on the vibratory motions, respectively. Karimi and Yazdanpanah [8] investigated a new methodology based on the singular perturbation method for modeling a single-link flexible manipulator. They showed that a part of the fast dynamics of the singularly perturbed system representing flexibility is treated as a norm-bounded uncertainty. Ghayesh and Khadem [9] investigated free nonlinear transverse vibration of an axially moving beam in which rotary inertia and temperature variation effects have been considered. They applied the multiple scales method to obtain steady-state response in equations of motion. Elimination of secular terms will give us the amplitude of vibration. They analyzed the stability of steady-state responses using RouthHurwitz criterion. To show the effects of rotary inertia, nonlinear term, temperature gradient and mean velocity variation, on natural frequencies, critical speeds, bifurcation points and stability

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of trivial and non-trivial solutions, they performed numerical examples. Tang et al. [10] analyzed nonlinear vibrations of axially moving beams based on the Timoshenko model under weak and strong external excitations. The nonlinearity caused by finite stretching of the beams. To obtain the transverse vibration modes and the natural frequencies of the linear equation, the complex mode approach is applied. They demonstrate the effects of a varying axial speed, external excitation amplitudes, and nonlinearity on the response amplitudes for the first and second modes by employed the method of multiple scales. Chen and Zhao [11] investigated Free nonlinear transverse vibration of axially moving beam modeled by an integro-partial-differential equation with a low axial speed. Chen and Yang [12] considered an axially moving viscoelastic, Euler-Bernoulli beam with time variant velocity. They used only strain which is caused by bending moment and neglected strain which is made by gradient of longitudinal displacement. Kartik and Wickert [13] investigated forced vibration of axially moving strip which is guided by a partial elastic foundation and edge imperfection. In the present investigation, a non-linear beam with mean velocity variation effects is considered. The speed is time dependent in translational and rotational motion, and the obtained equation is to form a partial differential equation. Applying multiple scales method, stability and bifurcation for frequency of variable transporting speed are investigated using Routh-Hurwitz criterion. Numerical examples show the effect of non-linear term and mean velocity on natural frequencies, bifurcation points, and stability.

Dehgolan et al. [14] investigated linear frequencies and stability of a flexible rotor-disk-blades system. Using EulerBernoulli beam theory, they considered the effects of various system parameters on the natural frequencies and clarified the decay rates (stability condition).

II. EQUATIONS OF MOTION

A beam with axial stiffness of *EA* and the flexural rigidity of *EI* is shown in Fig. 1. Additionally, this beam is assumed as an Euler–Bernoulli beam. The prismatic joint is assumed to be rigid and the flexible arm slides in the prismatic joint. The mass and flexible properties are considered to be distributed uniformly along the flexible arm, and the sliding motion of the flexible manipulator is assumed to be frictionless. The initial length of the beam is denoted as □ and a harmonically varying transport speed, v. As shown in Fig. 1, $w(x,t)$ describes transverse displacements of the beam.

It is obvious that kinetic energy is given by

l

T □ ‒ 1/2 □₀ □□□□(x□ ‒ □w □ □w□ x□□)² □ (x□□ w□□)² □ □ dx □ □x □ (1)

□ 1/2 m_e □□□(L□ □w L t□(x,) □□w L t(□t ,) □ L□□)² □ (L□ □ w L t(,))□□ ²□□□

in which □ is the constant mass per unit length, and m_e is the tip mass. Non-linear strain is used in order to calculate potential energy. Then, the non-linear strain and potential energy are found as

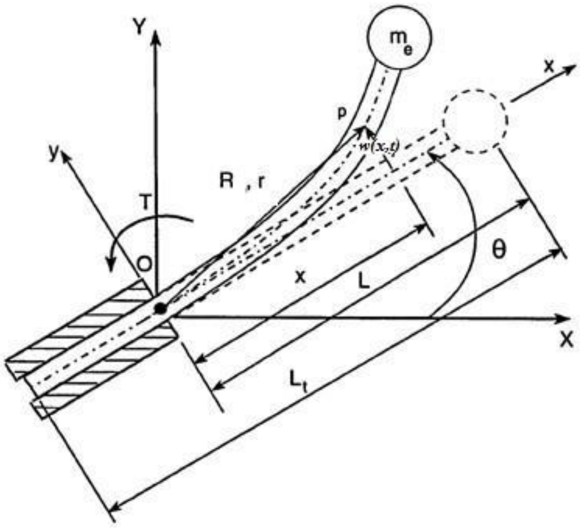

Fig. 1 Axially moving beam with rotating prismatic-joint

y □ ²xw₂ □ 12(□□wx)₂ □ₓₓ □ □ (2)

V □ 1/2 ∫₀ˡ EI(□²w/□x²)² dx □ 1/8 ∫₀ˡ EA(□w/□x)⁴ dx □ 1/4 ∫₀ˡ (L □x)(□□□²)(□w/□x)² dx

L□□ ₀L x□)(□w)₂dx□ 1m Lₑ(□□₂□L□□) (□₀ₗ □□wx)₂dx (3)

□ □ □(□x 2

2

The governing partial-differential equation and the associated boundary conditions are derived from the Hamilton's principle and the geometrical relations as

□(T □U dt) • □o (4)

t_2 t_1

Introducing dimensionless quantities

y □ w/L z □ x/L T □ t √(EI / • AL⁴) v □ LT v (5)

□₁□___□mALᵉ □₂□ ___ALI₂ □ □
□□□ □□□ □T □T □ (6)

Using (4), after simplification, the coupled non-linear equations would be

— □□v □ y □□2v□□y □□
□□t □□₂□□ □t □□z□□□□₂□□ □□□vt
□□□□□□yz □□□²t₂y □
□ — — — —

□v₂ □1□ z□₂□ □□1 z □□v □□² □1□ z₂□□□
□₁□□ ₂□□vt □□□□□□□□z²y₂ (7)
□□ t 2 □ □
□

□2 1v□ □ z□ □₂y □□₄y₄ □3□□₂ □□₂y₂ □□□y □₂□
z □□□2v□□o
□ □z t□z 2□□□z □□□□□z □□ □t

y(0,t) □o & ____□y(0,t) □o (8)

□z

•²y(1,t) ______ •₃y(1,t)
______• 0 & (9)
•z₂ • 0
•z₃

To use the multiple scales method, the non-linear term must be weak. Then, using transformation w □ □u and its substitution into (7), one obtains

□□v □

□ __t □□₂□u □□2v□__□ut
□□□□z□□□□₂□□ □□□vt □□□□□ □uz
□□□²tu₂
□ □ □ □

□2 1
v□ □ z□□ □□z t²u □□□z⁴u₄□ 32□□₂
□□□□□z²u₂ □□□□□□□□uz □□□² (10)

□ v₂1□ z□₂□ □□1 z□□□v □□₂□1□ —
z₂□□□ □₁□□ ₂□□vt
□□□□□□□□z₂u₂t 2
□ □
□□
□2v
□
□t

u(0,t) □o & ____□u(0,t)
□o (11) □z

_____□₂u(1₂,t) □ o
&_____□₃u(1₃,t) □ o (12)
□z □z

in which □ is a very small parameter (namely □□□1). As mentioned above, the beam is moving with a harmonically varying velocity about a constant mean velocity, i.e.

v □ □v₀ □v₁sin□₁t □ □ □□□ □₀ ₁sin□₂t (13)

□₁&₂ is the frequency of varying speed, v_0 is the mean translational velocity, □₀ is the mean rotational velocity, □v_1 is the amplitude of translational velocity, and □□ is the amplitude of rotational velocity. In order to find an approximated solution in a finite dimensional function space, the Galerkin method is used in this study. The solution of (7) is approximated by a series of comparison functions that satisfy both the essential and natural boundary conditions. The trial function for the approximated solution may be expressed as

$_N u\ z\ t$□ , □□ □ □□□$_n$ □

□$z\ q_n(\ )t$ □□ (14)

n□1

where N is the total number of comparison functions, $q\ t_n($) are the unknown functions of time to be determined, and □( )z are the eigenfunctions for the bending vibration of the stationary cantilever beam.

□ sin□$_n z$□sinh□$_n z$ □

□( )z □c_1 □□□sin□$_n z$□sinh□$_n z$□□ ________

cos□$_n z$□cosh□$_n z$□□cos□$_n z$□cosh□$_n z$□□□(15)

The weighting function or the virtual function corresponding to (14) is given by

□

u□$x\ t$, □□ □ □□□$_n$ □ □$z\ q_n(\ )t$ □□

(16)

n□1

Discretized equations of motion are determined by using (14) and (15). Consider an equation obtained by substituting (14) into (10), multiplying the resultant equation by (16) and then integrating it over the domain o □ □x 1, If this equation is collected with respect to $q(\ )t$, their coefficients provide the discretized equations since $q($ $)t$ are arbitrary. The discretized equations of axially moving beam with rotating prismatic joint may then be expressed as (17)

$^2A_n q_n$ □2$v\ A\ q_n$ □$_n$ □(□^{2} — □□v)□$_N B\ q_{nm}$ $_n$□ A — q_n□□$_n$ □v^2□$_N C\ q_{nm}$ $_n$ □□v □$_N D$ q_{nm} $_n$ □

(□□)

□t m□1 m□1 □t m□1

N v N N N

)□ $E\ q_{nmn}$ □□□$_1$($_2$□__□)□ $F\ q_{nmn}$ □2^v□G q_{nm} □$_n$ □□ $H_{nm}q_n$ □ _3□□^{2} □$_N$ $I_{nm}q_{n3}$ □ (17)

m□1 □t m□1 m□1 m□1 2 m□1

□□

□t □2K_n□v □ 0

where the superposed dot represents the differentiation with respect to time; are given by

0 1 1 n 0 1 2 1 n n 1 1 1 0 1 2 n n0

·q N 2

· ·2$v\ A_0$ $_n$· ____$_{n0}$ · ·2 $A\ v_{n1}$ sin · $_1 t$ ·q˙$_{n0}$ · ·v_1· $_1$ cos · $_1 t$ · 2··$_0$ $_1$ sin · $_2 t$ ·· $B_{nm}\ q_{n0}$ · 2 A_n____· q_{n0} ·

·t_1 m·1 · ·$t_0\ t_1$ (21)

N N N

· ·2$v\ v_{01}$ sin · $_1 t$ ·· $C_{nm}\ q_{n0}$ · ·v_1· $_1$ cos · $_1 t$ ·· $D_{nm}\ q_{n0}$ · ··$_0$ $_1$ sin · $_2 t$ ·· $E_{nm}\ q_{n0}$ ·

m·1 m·1 m·1

·$_N$ ·$_N$ ____ q_{n0} v_1 sin · $_1 t$

··$_N$ $G_{nm}\ q_{n0}$ ·

· ·v_1· $_1$ cos · $_1 t$ · 2··$_0$ $_1$ sin · $_2 t$ · $F_{nm}\ q_{n0}$ · ·2v_0 · G_{nm}˙ · ·2

m·1 m·1 ·t_1 m·1

N N

·· $H_{nm}\ q_{n0}$ · __3 ·$_2 q_{n30}$ · ··$_1$· $_2$ cos · $_2 t$ · J_n · __3 · ·$_{22}$ ·4 J_n · 6 L_n · 2 M_n ·· __3 ·$_2$ · $I_{nm}\ q_{n30}$ $_{m\cdot1}$ 2 2 2 $_{m\cdot1}$

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$$A_n = \int_0^1 \phi_n^2 \, dz \quad B_{nm} = \int_0^1 z \phi_n \phi_m' \, dz \quad C_{nm} = \int_0^1 (1-z)^2 \phi_n \phi_m'' \, dz \quad D_{nm} = \int_0^1 (1-z) \phi_n \phi_m'' \, dz$$

$$E_{nm} = \int_0^1 (1-z^2) \phi_n \phi_m'' \, dz \quad F_{nm} = \int_0^1 \phi_n \phi_m'' \, dz \quad G_{nm} = \int_0^1 (1-z) \phi_n \phi_m' \, dz \quad H_{nm} = \int_0^1 \phi_n \phi_m^{(4)} \, dz$$

$$I_{nm} = \int_0^1 \phi_n \phi_m'' (\phi_m')^2 \, dz \quad J_n = \int_0^1 z \phi_n \, dz \quad K_n = \int_0^1 \phi_n \, dz \quad L_n = \int_0^1 z^2 \phi_n \, dz \quad M_n = \int_0^1 z^3 \phi_n \, dz \tag{18}$$

$$N_n = \int_0^1 (1-z) \phi_n \, dz \quad O_n = \int_0^1 (1-z)^2 \phi_n \, dz \quad P_n = \int_0^1 (1-z)^3 \phi_n \, dz \quad Q_n = \int_0^1 (1-z)(1-z^2) \phi_n \, dz$$

Note that the dimensionless natural frequency of the stationary cantilever beam ω_n is equal to the square of the root of λ_n.

III. Stability and Bifurcations

Analytical methods often easily delineate general phenomena, yielding useful results in closed form [16]. The simple asymptotic expansions often fail to correctly result in appropriate solutions for problems which have secular terms. Using the method of multiple scales and assuming the solution to be a function of multiple independent scales of time, this method leads to a set of equations in different orders. Elimination of secular terms from these equations provides solutions. For more general form of the multiple scales method, see [15]-[17].

In perturbation method, $u(t, \varepsilon)$ is generally assumed as an asymptotical expansion.

$$u(t;\varepsilon) = u_0(T_0, T_1) + \varepsilon u_1(T_0, T_1) + \dots \tag{19}$$

in which $T_0 = t$ and $T_1 = \varepsilon t$. Substitution of (14) and (19) into (10) shows that

$O(\varepsilon^0)$:

$$\begin{aligned}
&(1+\alpha_0^2) A_n \ddot q_{n0} + 2v A_n \dot q_{n0} + (1+\alpha_0^2) \sum_{m=1}^{N} B_{nm} q_{m0} + A_n \ddot q_{n0} + v_0^2 \sum_{m=1}^{N} C_{nm} q_{m0} + \\
&+ \left(\frac{\alpha^2}{2} - \alpha_0 \right) \sum_{m=1}^{N} E_{nm} q_{m0} + (1+\alpha_0^2) \sum_{m=1}^{N} F_{nm} q_{m0} + 2v_0 \sum_{m=1}^{N} G_{nm} \dot q_{m0} + \sum_{m=1}^{N} H_{nm} q_{m0} = \\
&= -2K_n \dot\alpha_0 v_0 - \alpha_0 (1+\alpha_0^2) \left(\frac{M_n}{3} - \frac{L_n}{2} \right) + 2v P_{0} \dot\alpha_0^2 Q_n + (1+\alpha_0^2) N_n
\end{aligned} \tag{20}$$

$O(\varepsilon^1)$:

$$\begin{aligned}
&(1+\alpha_0^2) A_n \ddot q_{n1} + 2v A_n \dot q_{n1} + (1+\alpha_0^2) \sum_{m=1}^{N} B_{nm} q_{n1} + A_n \ddot q_{n1} + v_0^2 \sum_{m=1}^{N} C_{nm} q_{n1} + \\
&+ \left(\frac{\alpha^2}{2} - \alpha_0 \right) \sum_{m=1}^{N} E_{nm} q_{n1} + (1+\alpha_0^2) \sum_{m=1}^{N} F_{nm} q_{n1} + 2v_0 \sum_{m=1}^{N} G_{nm} \dot q_{n1} + \sum_{m=1}^{N} H_{nm} q_{n1} =
\end{aligned}$$

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· · 2 K n •v0•1 sin · 2t · v1•0 sin · 1t •· ˙ ..˙(v1· 1 cos · 1t · 2••0 1 sin · 2t)(____2 M3 n · Ln) ·

□ 4(v v sin □ t P) □ (□□ sin □ t)(2□ N □ Q)□□ □v □ cos □ t □ 2□□ sin □ t □ A q □

Assuming the solution of (20) as [14] in which □n is the natural frequency, and □n 1( )t is the

qon □t to 1, □ □□n □t1 □ei□nto □□_n □t1 □e□i□nto (22) amplitude. Substitution of (22) into (21) shows that

O □□1□ □

(□□o2)A qnm1 □ 2v A qo n n1 □ (□o2)□ $\sum_{m□1}^{N}$ Bnm qn1 □ A qn □□n1 □ vo2 □ $\sum_{m□1}^{N}$ Cnm qn1 □

□(□___o2 / 2)□ $\sum_{m□1}^{N}$ Enm qn1 □□□1(o2)□ $\sum_{m□1}^{N}$ Fnm qn1 □ 2vo □ $\sum_{m□1}^{N}$ Gnm q□n1 □ □ $\sum_{m□1}^{N}$ H nm qn1 □

□ □2K n □vo 1□ sin □2t □ v1 o□ sin □1t□ □ □□□□(____2M3 n □ Ln)(v1 1□ cos □1t □ 2□□o 1 sin □2t)□□□ □

□4(v vo 1 sin □1t P) n □ (□□o 1 sin □2t) 2□ □1N n □ Qn □□ □□□v1 1□ cos □1t □ 2□□o 1 sin □2t□□

□An (□n □ □t1 ei□nto □ qnpo)□□ □ □2v Ao n □□□n□ □ □t1 ei□nto □ □ □2A vn 1 sin □1t□ □□n □ □t1 i□nei□nto □ □

□ □v1 1□ cos □1t □ 2□□o 1 sin □2t□ □ $\sum_{m□1}^{N}$ Bnm (□n □ □t1 ei□nto □ qnpo) □ 2An □□n□ □ □t1 i□nei□nto □

· □□□2v vo 1 sin □1t□ □N Cnm □ □v1 1□ cos □1t□ □N Dnm □ □□□o 1 sin □2t□ □N Enm □□ (□n □ □t1 ei□nto □ qnpo) □

□ m□1 m□1 m□1 □ m□1 m□1 2

· N N (23)

· ·•v1 1· cos ·1t · 2··0 1 sin ·2t•· Fnm · ·2v1 sin ·1t•· Gnm ·· (·n · ·t1 ei·nt0 · qnp0) ·

· m·1 m·1 ·

N N

· ·2v0 •· Gnm •·n· · ·t1 ei·nt0 • · · H nm (·n · ·t1 ei·nt0 · qnp0) · _3· ·2(n · ·t1 ei·nt0 · qnp0)3 ·

· □□1□2 cos □2t J□ n □ _3□□2 2 □4J n □ 6Ln □ 2M n □ □ cc □ h o t. .

2

in which, when □1 is close to 2□n, subharmonic resonance where □ is the detuning parameter, the solvability condition can be obtained using (21) as

will occur. Let us consider

□ □12□n □□□ (24)

d□n

2 __ei T□ 1 □0 (25)

dt1 □□□□ □ □ □ □n1 n □□ n2 □ n □□ n1 □ n

in which

: 2 2 • nm : □ 9 N I □□ 9□2 2p □ □N Inm □ N

m· 1 Hnm n□ □□ q n0

□n1 □ N □n2 □ □ 2 m□1 m□1 N □

□ □ □ □

□2□An □v0 □ i□n □ □ □□ v0 □ Gnm □□2□An □v0 □ i□n □ □ □□ v0 □ Gnm □

□ m□1 □ □ m□1 □

□□____□v1 1□ □□□□ An □ □ □ □N Bnm □ N Dnm □ N Fnm □□□□ □ □□iv1 □□ An □ v0 □ □N Cnm □

N Gnm □□□□ (26)

□n □ □2 □ m□1 m□1 m□1 □□ □ □ m□1 m□1 □□

1 □ N □

□2□ An □v0 □ i□n □ □ □□ v0 □ Gnm □

□ m□1 □

Let

$$□_n(T_1) □ \frac{1}{2} a_n(T_1) e^{i□_n(T_1)} \quad (27)$$

Using (23) and (25), one has (28)

□a^n □□ ^{1} Re(□ $_1$ a_n^3 □ Re□ □$_{n2}$□a a_n □ $_n$ □Re(□$_{n1}$)cos(□$_n$) □ Im(□$_{n1}$)sin(□$_n$)□

(28) $_n$)

□T_1 4

□□$_n$ □ □□ ^{1} Im(□^{n})a_n^2 □ 2 Im □ □$_{n2}$ □□ 2□Im(□$_{n1}$) cos(□$_n$) □ Re(□$_{n1}$) sin(□$_n$)□ (29)

$_1$

□T_1 2

in which

□$_n$ □□ □T_1 □ $_n$

□□$_n$ □□$_n$

□ □□ (30)

As one considers the stationary response, the value of a_n□ and □$_n$□ will be equal to zero. Elimination of □$_n$ between (28) and (29) leads to

(31)

Using (28) and (29) and constructing the Jacobian matrix,

J □□I □0 (32)

Re(□$_{n1}$)a_{n2} □Re□ □ □$_{n2}$□□ □□ a_n□Im(□$_{n1}$)□ □□

□ □ (33)

1 $a^n{}_2$ □2Im□□$_{n2}$□□□□□□ □□□ 1^2 Re(□$_{n1}$)$a^n{}_2$ □2Re□□$_{n2}$□□□□□□□□□

Im(□$_n$) | |

2 $_1$ □ □

From (31) and (33), one has

□$_2$ □□□1a_{n4} Re(□$_{n1}$)$_2$ □ 3a_{n2} Re(□$_{n1}$)□Re□□ □$_{n2}$□□□ □

□8 4 □

(34)

□□a_n □Im(□$_{n1}$)□□□□□□□□a2$_n$ □□□□□ 12Im(□$_{n1}$)a_{n2}

□2Im□□$_{n2}$□□□□□□□□□□□0

□□

From (31) and (34) and using the Routh-Hurwitz criterion, Fig. 4. When □□□$_1$, there is a stable trivial solution. At □□□$_1$, the stability condition can be obtained as below the trivial solution starts to be unstable, and a stable nontrivial solution bifurcates. At □□□$_2$, the trivial solution starts to be

a Re(□$_{n1}$)^2 □ $\frac{3}{4}$ a_n^2 Re(□$_{n1}$) □ Re □□$_{n2}$ □□□ □ 0 (35)

Im(□$_{n1}$)a_{n2} □ 2 Im □□$_{n2}$ □□□

□□a_n □Im(□$_{n1}$)□□□

IV. SIMULATION

In this section, the objective is to study natural frequencies according to mean velocity. Also, the effects of non-linear term, mean velocity on stability are investigated. In the other words, one would like to assess how the natural frequencies, stability, and bifurcation points will change when system parameters change. Figs. 2 and 3 show that increasing the time would lead to a reduction in first two natural frequencies of system.

V. CONCLUSION

In this section, numerical simulations are presented to show the effectiveness of the analytic method. Frequency-response curve of the system which is governed by (34) is depicted in

stable again, and then unstable, nontrivial solution appears. It means that the bifurcation point will appear sooner. Through (35) and numerical simulations, it can be concluded that for the dynamic model, the curve first detuning parameter □$_1$ is □$_2$ always stable, and the curve of second detuning parameter is always unstable.

In Figs. 5-7, when □□□$_1$, only stable trivial solution exists. When □□□$_1$, the trivial solution will be unstable, and a stable nontrivial solution occurs. When □□□$_2$, the trivial solution starts to be stable again, and an unstable nontrivial solution occurs. In Figs 5-7, at ···$_1$, a stable trivial solution exists. When ···$_1$, the trivial solution starts to be unstable, and an unstable nontrivial solution occurs. At ···$_2$, the trivial solution starts to be stable again, and an unstable nontrivial solution bifurcates. Increasing "· " leads to a smaller instability interval for trivial solution.

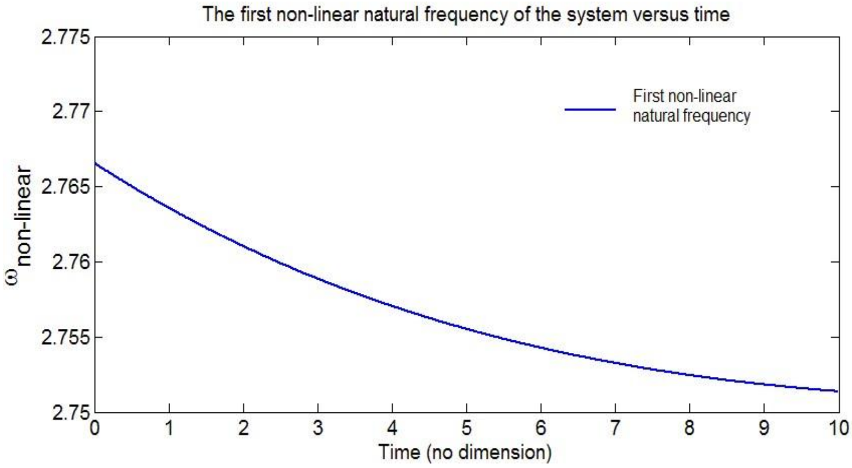


Fig. 2 First natural frequency versus the mean velocity and rotary inertia for the first two modes

v_0□ 0.072;v_1□ 0.001;□$_0$□ 2.6;□$_1$□ 0.002;□$_{n1}$□ 0.16;□$_{n2}$□15.2;□$_{n1}$□16.82

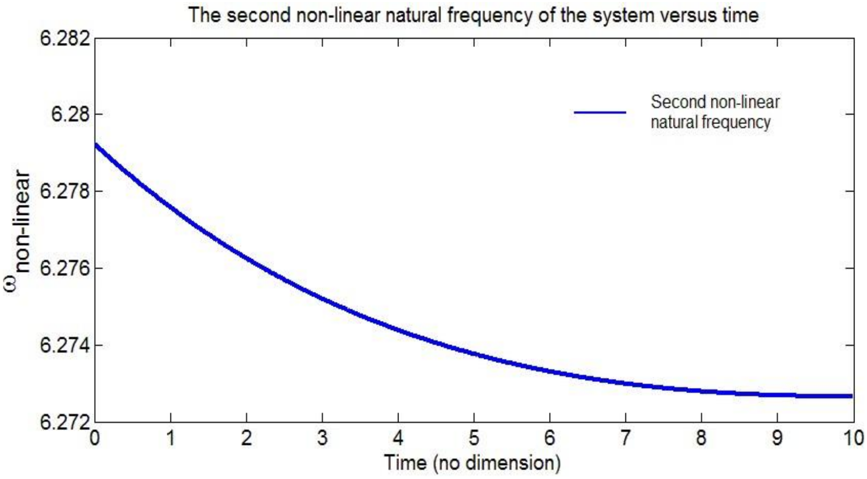


Fig. 3 Second natural frequency versus the mean velocity and rotary inertia for the first two modes
v_0 □ 0.072;v_1 □ 0.001;□$_0$ □ 2.6;□$_1$ □ 0.002;□$_{n1}$ □ 0.16;□$_{n2}$ □15.2;□$_{n1}$ □16.82

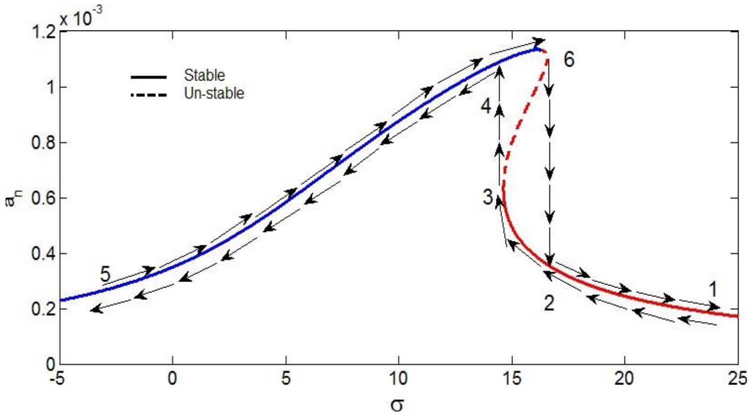
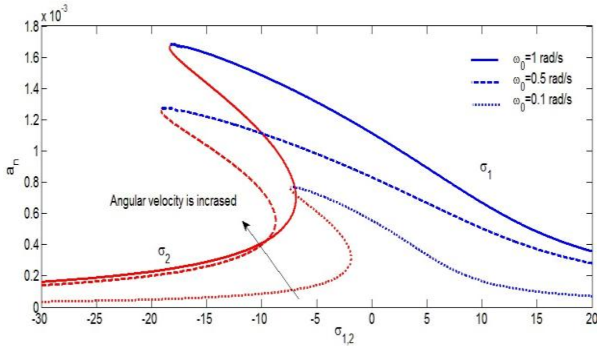


Fig. 4 Stability and bifurcation points' variation for the first mode (dashed line: unstable and solid line: stable) v_0 □ 0.072;v_1 □ 0.001;□$_0$ □ 2.6;□$_1$ □ 0.002;□$_{n1}$ □ 0.16;□$_{n2}$ □15.2;□$_{n1}$ □16.82

International Science Index, Mechanical and Mechatronics Engineering Vol:11, No:4, 2017 waset.org/Publication/100663

3

0 1 $n1$



0 1 $n1$

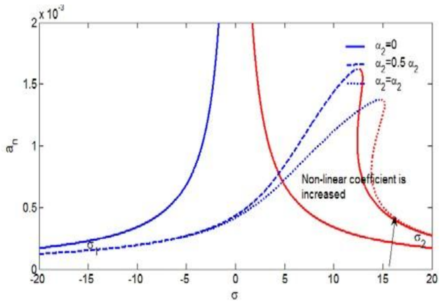


Fig. 7 Stability and bifurcation point variation under the non-

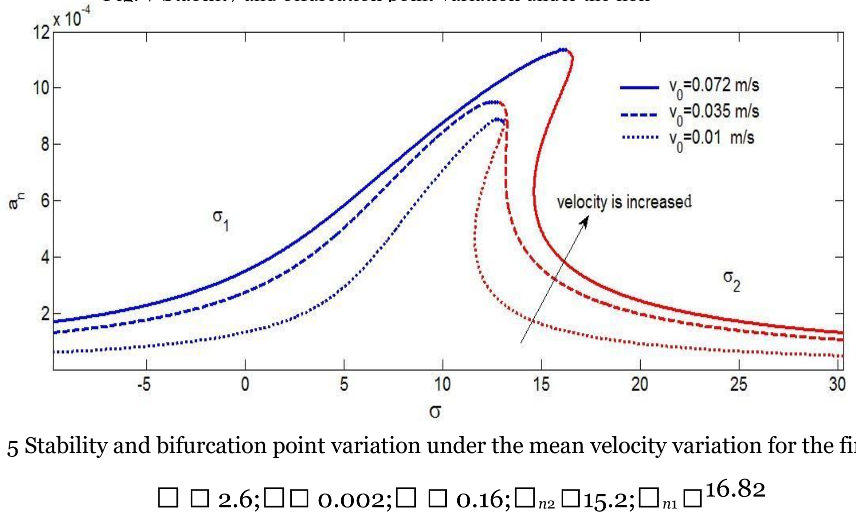


Fig. 5 Stability and bifurcation point variation under the mean velocity variation for the first mode

□ □ 2.6;□□ 0.002;□ □ 0.16;□$_{n2}$ □15.2;□$_{n1}$ □16.82

Fig. 6 Stability and bifurcation point variation under the mean angular velocity variation for the first mode υ □ 0.072;υ □ 0.001;□ □ 0.16;$□_{n2}$ □15.2;$□_{n1}$ □16.82

term variation for the first mode

υ_0 □ 0.072;υ_1 □ 0.001;$□_0$ □ 2.6;$□_1$ □ 0.002;$□_{n1}$ □ 0.16;$□_{n2}$ □15.2;$□_{n1}$ □16.82

V. Stability under Variation of the Mean Translational Velocity and Non-Linear Term

Free non-linear vibration of axially moving beam with rotating prismatic joint in which non-linear strain have been considered was investigated. The beam is moving under constant a mean translational and rotational velocity with Decreasing mean translational and rotational velocity led to a reduction in stability of system, but increasing mean angular velocity made stability increased.

small periodic fluctuations. A time-dependent partial differential equation and boundary conditions are derived to describe the beam lateral deflection. The multiple scales method was used. It was realized that when speed fluctuation frequency is close to twice of the natural frequency, the principal parametric resonance would arise. For this case, the effects of mean translational velocity and mean rotational velocity on natural frequencies were investigated. Stability and bifurcation of non-trivial and trivial steady state responses were analyzed by using Routh-Hurwitz criterion. The effects of non-linear term and mean velocity on bifurcation points and stability of trivial and non-trivial solutions also were investigated, and the frequency-response curves were drawn.

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