Journal of Computer Engineering
ISSN 10003428

Original Research Article
Vol 9, Issue 3, 2020 | ISSN: 1000-3428

Optimizing Joint Source-Channel Coding for Correlated Sources Transmission over Noisy Channels

Dr. Fatima El Mehdi, Dr. Omar Khaled, Dr. Leila Benkirane

Dr. Fatima El Mehdi and Dr. Omar Khaled, Department of Electrical Engineering, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia; Dr. Leila Benkirane, Signal Processing and Communications Laboratory, University of Mohammed VI Polytechnic, Benguerir, Morocco

Abstract—In this paper, a Joint Source Channel coding scheme based on LDPC codes is investigated. We consider two concatenated LDPC codes, one allows to compress a correlated source and the second to protect it against channel degradations. The original information can be reconstructed at the receiver by a joint decoder, where the source decoder and the channel decoder run in parallel by transferring extrinsic information. We investigate the performance of the JSC LDPC code in terms of Bit-Error Rate (BER) in the case of transmission over an Additive White Gaussian Noise (AWGN) channel, and for different source and channel rate parameters. We emphasize how JSC LDPC presents a performance tradeoff depending on the channel state and on the source correlation. We show that, the JSC LDPC is an efficient solution for a relatively low Signal-to-Noise Ratio (SNR) channel, especially with highly correlated sources. Finally, a source-channel rate optimization has to be applied to guarantee the best JSC LDPC system performance for a given channel.

I. Introduction

IN information theory, Shannon showed that reliable transmission is possible through a noisy channel if the source and channel coding are examined separately [1]. The separation theorem was demonstrated for stationary channels and sources with infinite lengths. However, in real life wireless communication systems, finite-block lengths are used for favorable performance, and the channels are not necessarily stationary. Therefore, we should be careful before employing the separation design, because it is possible that we cannot adjust quickly to the source and channel codes [2]. Furthermore, the separation theorem is not accurate in some cases such as multiuser channels [3]. All these issues made the Joint Source and Channel coding (JSC) an interesting topic for research.

The general idea behind the JSC approach consists of exploiting the residual redundancy left by the source code to enhance the performance of the communication system [4].

In this context, many contributions consider entropy encoding

techniques with iterative decoding attached on residual source redundancy [5]-[8]. An undesired effect of the entropy coding is to render the data transfer process more susceptible to channel noise. Also, standard fixed-to-variable length source codes (e.g. arithmetic codes) are not adapted for channel codes (e.g. LDPC or turbo codes) that use long sequences to reach the entropy of the source and even single error cause desynchronisation and error propagation in the source data [9]. Other contributions used JSC methods exploiting the correlation of the source without any compression schemes to improve the system performance such as [10]-[12]. However, for bandwidth limited wireless systems, the exploitation of the non-compressed source correlation is not practical. A solution to these problems mentioned above proposed in [9] is the use of a double LDPC code used to have fixed-to-fixed source and channel coding. The new structure of JSC coding is provided by a fixed length source encoder using an LDPC code to compress a redundant source, followed by a conventional LDPC channel code to protect the compressed source against transmission errors.

In this paper, we investigate the JSC LDPC-based coding system for a point-to-point AWGN channel with a joint decoder and correlated sources. The JSC decoder considered in this work has a single bipartite graph, decoded with an iterative decoding algorithm, such as belief propagation (BP). The main objective behind this work is to emphasize the improvements we can reach by applying joint LDPC in different source and channel contexts. Hence, as a first contribution, we showed that the JSC LDPC can be a good solution for highly disturbed channels and highly correlated sources. Second, we demonstrated that the source and the channel coding rates should be allocated according to the source correlation and to the channel state. We finally, illustrate that the JSC LDPC scheme suffers from a residual error demonstrated by a BER error floor that mainly depends on the source correlation and on the source coding rate.

The rest of the paper is organised as follows. In Section II, we introduce the system model. We present the JSC iterative decoding based on LDPC codes and correlated sources in Section III. In Section IV, simulation results and discussions are treated. Finally, Section V concludes the paper.

II. System Model

How to cite: Dr. Fatima El Mehdi, Dr. Omar Khaled, Dr. Leila Benkirane (2020). Optimizing Joint Source-Channel Coding for Correlated Sources Transmission over Noisy Channels. Journal of Computer Engineering, 9(3), 68–74. DOI: https://doi.org/10.5281/zenodo.19341520

In this paper, we consider a JSC LDPC coding to transmit a correlated source modelled by a two-states Markov process,

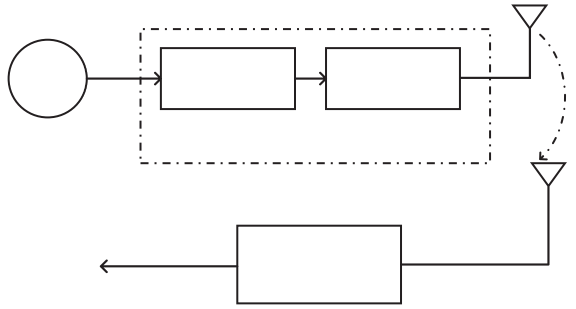

Fig. 1 The point-to-point reference system using JSC LDPC coding

through an AWGN channel. We refer the sequence generated by the source s as $(s_1,s_2,...,s_n)$ with n symbols length. The transition probabilities are expressed by $\alpha = \Pr(S_t = 1|S_{t-1} = 0)$ and $\beta = \Pr(S_t = 0|S_{t-1} = 1)$. The entropy rate of the stationary binary Markov source is provided by $H(S) = \mu_0\alpha\log(\alpha)\ \mu_0(1\ \alpha)\log(1\ \alpha)\ \mu_1\beta\log(\beta)\ \mu_1(1 - \beta)\log_2(1-\beta)$, where $\mu_0 = \Pr(S_t = 0)$ and $\mu_1 = \Pr(S_t = 1)$ are the stationary state probabilities.

We also use a double LDPC code [9] for JSC coding. The particularity of joint LDPC code is that both LDPC source and LDPC channel codes are concatenated in a structure described as follows. The first code compresses the data source and the second one protects the data. First, we compress the source using the $(l \times n)$ parity check matrix of the source code H_{sc} by

$$\mathrm{b} = \mathrm{H}_{sc} \times \mathrm{s}. \tag{1}$$

The column vector s presents the n bits generated by the source, and b is the compressed sequence which has l bits. Second, we protect the compressed sequence with another LDPC code as:

$$\mathrm{c} = \mathbf{G}_{cc}^T \times \mathbf{b} = \mathbf{G}_{cc}^T \times \mathbf{H}_{sc} \times \mathbf{s}, \tag{2}$$

where c is the codeword to be transmitted with m bits length, and G^T_{cc} is the $(l \times m)$ generator matrix. We notice that H_{cc} is the $((m-l)\times m)$ parity check matrix of the channel code.

The overall rate is described by $R = \frac{l/m}{l/n} = n/m$ for the JSC encoder. The output of the JSC encoder is modulated by binary-phase shift keying (BPSK) and then transmitted over AWGN channel. At the decoder side, we consider a joint decoder to estimate the original sequence, based on the Log-Likelihood-Ratios of the transmitted codeword c, LLR(c), as depicted in Fig. 1. The joint decoder is represented in Fig. 2 as a single graph, where the Tanner graph of the source and the channel codes run in parallel. More details about iterative decoding are provided in the next section.

III. JSC Iterative LDPC Decoding

As mentioned, we consider a joint decoder that uses the BP iterative algorithm. The graphical model of the JSC decoder is composed by two Tanner graphs. Each graph is represented by two types of nodes, which are the variable and check nodes corresponding respectively to the columns and rows of each parity check matrix of the LDPC source and channel codes. As shown in Fig. 2, the solid lines connecting the l check nodes from the source code graph to the first variable nodes in the channel code graph, depicts codes concatenation. The joint decoder runs in parallel. First, the variable nodes v of the two decoders inform the check nodes c about their Log-Likelihood ratios (LLRs) for every k iteration. For $v = (1,...,n)$, we have:

$$m_{v,c}^{sc,(k)} = Z_v^{sc} + \sum_{c' \neq c} m_{c',v}^{sc,(k-1)} \tag{3}$$

where $Z_v^{sc} = \log(\frac{1-p_v}{p_v})$ are the LLRs for variable nodes, used to initialize the source decoder, where $p_v = \Pr(s_v = 1)$. Notice that the messages $m_{c',v}^{sc,(0)}$ are initialized by 0. Then,

$$m_{v,c}^{cc,(k)} = Z_v^{cc} + m_v^{sc\to cc,(k-1)} + \sum_{c' \neq c} m_{c',v}^{cc,(k-1)} \tag{4}$$

where $Z_v^{cc} = \frac{2r_v}{\sigma^2}$ represent the LLRs of the variable nodes for $v = (n + 1,...,l)$ of the channel decoder, $\mathrm{r} = (r_{n+1},...,r_l)$ is the noisy observed vector, n_v represents the AWGN noise sample and σ^2 is the channel noise variance. $m^{sc_v\to cc,(k-1)}$ represents the message transfer from the check nodes of the source decoder to the variable nodes of the channel decoder for $v = (n + 1,...,l)$. Initially, $m_{c',v}^{cc,(0)} = 0$ and $m_v^{sc\to cc,(0)} = 0$.

Also, the variable nodes of the channel decoder inform the check nodes of the source decoder about their LLRs for $v = (n + 1,...,l)$, as shown in the following expression

$$m_v^{cc\to sc,(k)} = Z_v^{cc} + \sum_{c'} m_{c',v}^{cc,(k-1)} \tag{5}$$

Then, for $v = (n + l + 1,...,n + m)$, the messages between variable v and check nodes c are illustrated by:

$$m_{v,c}^{cc,(k)} = Z_v^{cc} + \sum_{c' \neq c} m_{c',v}^{cc,(k-1)} \tag{6}$$

Now, we describe the messages between the check nodes c and the variable nodes v for the source decoder. For $c = (1,...,l)$, these messages are

$$m_{c,v}^{sc,(k)} = 2\tanh^{-1}\left(\prod_{v' \neq v} \tanh\left(\frac{m_{v',c}^{sc,(k)}}{2}\right) \tanh\left(\frac{m_v^{cc\to sc,(k)}}{2}\right)\right)$$

(7)

and

$$m_v^{sc \to cc,(k)} = 2\tanh^{-1}\Big(\prod_{v'} \tanh\Big(\frac{m_{v',c}^{sc,(k)}}{2}\Big)\Big) \quad (8)$$

For $c = (l + 1,\ldots,m)$, every check node c forwards to each connected variable node v of the channel decoder the message

$$m_{c,v}^{cc,(k)} = 2\tanh^{-1}\Big(\prod_{v' \neq v} \tanh\Big(\frac{m_{v',c}^{cc,(k)}}{2}\Big)\Big) \quad (9)$$

Finally, after k iterations of the decoder, we can estimate the source bits for $v = (1,\ldots,n)$ based on the a posteriori LLR:

$$\mathrm{LLR}(\mathbf{s}_v) = Z_v^{sc} + \sum_c m_{c,v}^{sc,(k)} \quad (10)$$

rate of $R = 1/2$. In both schemes, we apply 100 iterations of the BP algorithm. Fig. 3 shows the Bit-Error-Rate (BER) performance as a function of E_b/N_0 for the two configurations.

First, we can see that the JSC LDPC performance is better than the channel coding LDPC setup in the waterfall region. $\alpha = \beta = 0.07$ case achieves an improvement of about 4 dB for a BER equal to 10^{-5} compared to the same rate channel LDPC code. We observe the appearance of an error floor for high SNRs, which is due to the LDPC source coding which is not an optimized entropy source encoder. The error floor reach almost 3.10^{-6} for the 0.36 entropy case. Finally, we conclude that the JSC LDPC is a performance efficient solution for such a highly correlated source where, we can achieve a better error correction for highly disturbed channels at the cost of a very low error floor. Many real-life applications with correlated sources like multimedia broadcasting over wireless channels are bandwidth constrained but can tolerate low-rate residual

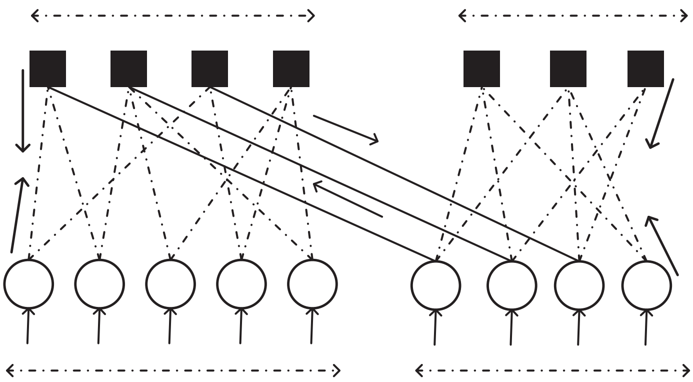

Fig. 2 The JSC decoder Tanner graph

errors. In this context, the JSC LDPC code can be a good alternative to the traditional separated coding approach. The main objective behind the next section is to investigate

IV. Simulation Results

In this section, we study the performance of the joint iterative decoding previously described by Monte-Carlo simulations. First, we compare the JSC encoding based on LDPC codes with respect to equivalent rate standard LDPC encoding to emphasize the efficiency of the JSC LDPC in the case of correlated sources. Second, we study the tradeoff between the waterfall and the error floor regions using the JSC LDPC coding for different system setup.

A. Joint Source Channel LDPC Coding

We consider a symmetric two states Markov source with transition probabilities α and β given by $\alpha = \beta = 0.07$ ($H(S) = 0.36$). We consider a point-to-point setup with regular LDPC codes. The reference is an LDPC channel code with $R = 1/2$ and $n = 3200$ bits. For JSC encoding, we use two concatenated source and channel LDPC codes with rates $R_s = 1/2$ and $R_c = 1/4$ respectively, which means also an overall

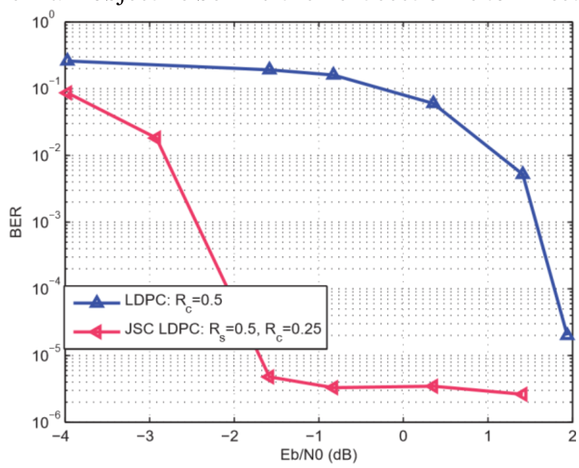

Fig. 3 BER performance for the point-to-point system with JSC LDPC rates: R_s =0.5 and R_c =0.25 and a joint decoder compared to the conventional LDPC channel decoder

the effects of the rate allocation between LDPC source and channel coding on the JSC LDPC code performance.

B. Effect of the Source and Channel Coding Rates

The idea behind the second simulation is to demonstrate the tradeoff, for a given source, between the error floor and the waterfall regions induced by JSC LDPC encoding.

We propose to study two configurations with almost the same overall coding rate R. The first code is composed of two concatenated source and channel LDPC codes with rates $R_s = 6/8$, $R_c = 6/9$ respectively, thus an overall rate of 0.88. The second one, has rates $R_s = 4/8$ and $R_c = 4/10$ respectively, which means an overall rate of 0.8. We can see that the first configuration make less compression at the expense of a lower protection capacity. However, the second configuration compresses more and allocates more bits for the correction LDPC code. Fig. 4 presents the BER as a function of E_b/N_0 for a symmetric Markov source with $\alpha = \beta = 0.07$.

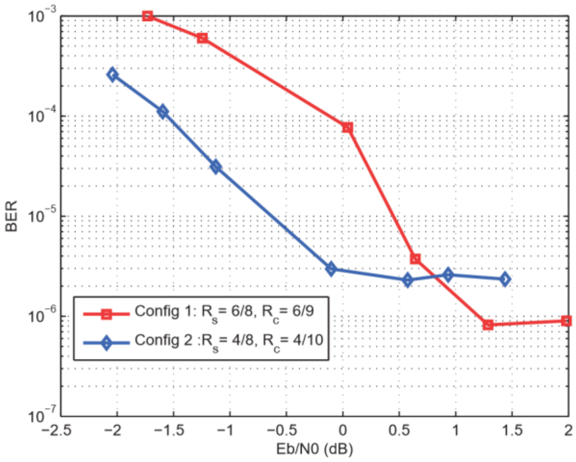

Fig. 4 BER performance for the point-to-point system with JSC LDPC for different source coding and channel coding rates

We observe that the first configuration that compresses less than the second one induces a reduced error floor with a BER equal to 10^{-6}. Such a behaviour is justified by the compression loss which depends mainly on the source coding rate. Using lower source coding rates involves higher error floors for a high SNR. On the other hand, the second configuration protects more with a channel coding rate $R_c = 4/10$ than the first one, consequently presents an improvement in the waterfall region with a gain of about 1 dB for a BER=10^{-5}. The waterfall region depends on the code correction capacity which is related to the channel coding rate. Lower channel coding rates involve more protection, hence improvement in the waterfall region. We conclude that the waterfall region performance is mainly depending on the channel code rate, although the error floor region is depending on the source code rate. Finally, we also notify that for a given source coding rate, the BER error floor value should also depend on the source correlation. The aim of the next section is to investigate this issue.

C. Effect of the Source Memory

Now, we study the effect of the source memory on the JSC LDPC. For this aim, we consider three symmetric and asymmetric cases with different transition probabilities: $\alpha = \beta = 0.07$ with $H(S) = 0.36$, $\alpha = 0.15, \beta = 0.25$ with $H(S) = 0.68$, and $\alpha = \beta = 0.1$ with $H(S) = 0.46$,
BER results for the three sources with respect to E_b/N_0 are prouded in Fig. 5. Compared to the other configurations, the performance of the JSC LDPC is the best with $\alpha = \beta = 0.07$. The BER is reduced at the expense of an error floor reached at an E_b/N_0 of almost -1.6 dB. The error floor is a result of the residual decoding errors introduced by the fixed-to-fixed source code. Moreover, the error floor depends on the source entropy. $\alpha = 0.15, \beta = 0.25$ source case induces a source rate which is less than the source entropy, then makes lossy compression which justifies the high error floor. Hence, we can conclude that the JSC LDPC coding can be a good solution under some specific asumptions. In fact, we can have

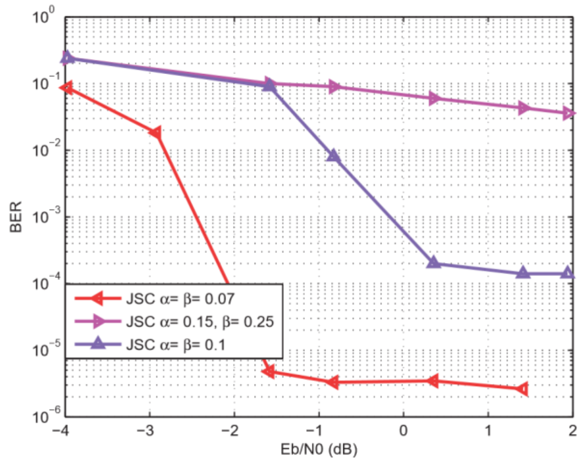

Fig. 5 BER performance with JSC LDPC system for different source distributions

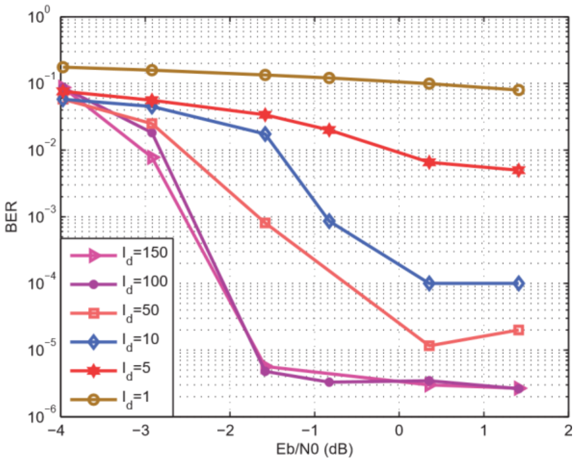

Fig. 6 Effect of the iterations number of the joint decoder on JSC LDPC performance

better improvements if the source is highly correlated and the channel is relatively very noisy. Then, the JSC LDPC approach should be provided with an efficient source and channel coding rates allocation to work for every system (source and channel) configurations.

D. Effect of the Iterations Number of the Joint Decoder

In this section, we propose a last experiment to study the JSC LDPC system convergence through iterations. In Fig. 6, we show the effect of the iterations number I_d of the joint decoder on the performance of JSC LDPC coding. We plot the BER as a function of E_b/N_0 for point-to-point system and Markov source where $\alpha = \beta = 0.07$.

We observe that the performance in waterfall and error floor regions improves with an increasing number of iteration. With $I_d = 1$, the BER does not improve and we have high error floor. $I_d = 100$ provides a gain of about 1.3 dB for BER=10^{-2} compared to $I_d = 10$ for the same system, and a gain of about

2.8 dB compared to $I_d = 5$. We can conclude that with the $I_d = 150$ performance that the BP algorithm is converging to a fixed value of the BER at the 100^{th} iterations. We can also see that the iterative process improves both the error floor and the waterfall BERs and this is due to the joint process. Having correct LLRs at the channel decoding process improves the LLRs of the source decoder, and reduces consequently the error floor.

V. Conclusion

In this paper, we studied a JSC coding based on two concatenated LDPC codes with a joint decoding process for point-to-point system and correlated sources. First, we demonstrated the performance of the JSC LDPC encoding and decoding scheme based on simulation results. We showed an improvement of about 4 dB compared to the LDPC channel decoder, where we decode the transmitted sequence without any source model. Second, we showed the tradeoff for a given source between the waterfall and error floor regions, using different source and channel coding rates. We noted that the waterfall region is depending on the channel code parameters, although the error floor is due to the source code setup. Then, we studied the effect of the source memory with different source distributions. We can conclude that with high correlated sources, the JSC LDPC code can achieve better performance for a highly noisy channel.

References

[1] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, pp. 379-423, 623–656, 1948.

[2] S. Vembu, S. Verdu, and Y. Steinberg, "The source-channel separation theorem revisited," *IEEE Trans. on Inform. Theory*, vol. 41, no. 1, pp. 44–54, Jan. 1995.

[3] T. M. Cover and J. A. Thomas, "Elements of Information Theory," John Wiley, New York, 1991.

[4] K. Sayood and J. C. Borkenhagen, "Use of residual redundancy in the design of joint source/channel coders," *IEEE Trans. on Comm.*, vol. 39, no. 6, pp. 838–846, June 1991.

[5] A. Zribi, R. Pyndiah, S. Zaibi, F. Guilloud, and A. Bouallgue, "Low-complexity soft decoding of Huffman codes and iterative joint source/channel decoding," *IEEE Trans. on comm.*, vol. 60, n. 6, pp. 1669–1679, Jun. 2012.

[6] R. Bauer and J. Hagenauer, "On variable length codes for iterative source/channel decoding," *Proc. Data Compression Conference*, pp. 273–282, April. 2001.

[7] J. Kliewer and R. Thobaben, "Iterative joint source–channel decoding of variable length codes using residual source redundancy," *IEEE Trans. Wireless. Commun.*, vol. 4, no. 3, pp. 919–929, May 2005.

[8] H. Nguyen and P. Duhamel, "Robust source decoding of varaible–length encoded video data taking into account source constraints ," *IEEE Trans. Commun.*, vol. 53, no. 7, pp. 1077–1084, Jul. 2005.

[9] M. Fresia, F. Prez-Cruz, H. V. Poor, and S. Verdu, "Joint source and channel coding," *IEEE Sig. Proc. Mag.*, vol.27, pp. 104–113, Nov. 2010.

[10] R. Asvadi, T. Matsumoto, and M. J. Juntti, "Optimized LDPC codes for joint source-channel decoding of quantized Gauss-Markov signals," *IEEE International Conference on Communications (ICC)*, pp. 5233–5238, 2014.

[11] T. Hindelang, J. Hagenauer, and S. Heinen, "Sources–controlled channel decoding: Estimation of correlated parameters," in *Proc. 3rd Int. ITG Conf. on source and channel coding*, pp. 259–266, Munich, Germany, 2000.

[12] M. A. Mohd Izhar, N. Fisal, X. Zhou, K. Anwar, and T. Matsumoto, "Exploitation of 2D binary source correlation using turbo block codes with fine-tuning ," *EURASIP Journal on Wireless Communications and Networking*, vol. 2013, no. 1, pp. 1–11, 2013.