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Geometric Transformations in Quantum Field Theory: Implications for the Spectral Analysis of Spacetime

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***Abstract*—**Through the Fukaya conjecture and the wrapped Floer cohomology, the correspondences between paths in a loop space and states of a wrapping space of states in a Hamiltonian space (the ramification of field in this case is the connection to the operator that goes from TM to T^*M) are demonstrated where these last states are corresponding to bosonic extensions of a spectrum of the space-time or direct image of the functor Spec, on space-time. This establishes a distinguished diffeomorphism defined by the mapping from the corresponding loops space to wrapping category of the Floer cohomology complex which furthermore relates in certain proportion D-branes (certain D-modules) with strings. This also gives to place to certain conjecture that establishes equivalences between moduli spaces that can be consigned in a moduli identity taking as spacetime the Hitchin moduli space on G, whose dual can be expressed by a factor of a bosonic moduli spaces.

I. INTRODUCTION

IN symplectic geometry, the Lagrangian submanifolds are invariants under rescaling of the cotangent fibers or more generally asymptotically invariant which determine diffeomorphisms between the said cotangent classes under homotopies, establishing a cohomology of Floer cohomology type where wrapping states are in correspondence with paths in certain based loop space. By the Fukaya conjecture, this correspondence can be carried to the equivalences context between categories whose objects are Lagrangian submanifolds of the complex Riemannian smanifold (complex model of the space-time), and whose morphisms are Floer chain groups of the type Hom(L_0,L_1) □ FC(L_0,L_1). Likewise, if L□T_x^*, has cotangent fiber then the $A_□$ □ structure on

$CW^*(L,L)$, should be quasi-isomorphic to the dg□ algebra structure on $C_{□□}$(□$_x$),where □$_x$,is the based loop space of (Z,x),[1]; that is to say, there is a minimal path from (Z,x), to
□
(L,x), taking a connection with ramification.

From a point of view of the physics, this could have implications in the process of explanation of the relation between movement and energy, problem planted from Huygens and Lagrange to the dynamical problems, which requires the integration of energy and movement through certain integral operators that can be induced to the micro-

local structure (fine structure) of the Lagrangian submanifolds to the QFT context using the TFT and some tools of cohomology to obtain the equivalences in duality of these Lagrangian objects. From aa algebraic point of view, using commutative rings, it can be induced to a scheme that establishes the relations between moduli problems and algebras involved in the reduction problem of field equations to some cases in field theory. Is it feasible to determine symmetry between energy and time? We can answer categorically no! However, realizing a re-evaluating of the question in the framework of the ramified field, could determine an equivalence, not in a purely algebraic context and the continuous mappings only, but insome differential applications that born of several dualities, as have been mentioned (for example, the Langlands duality [2], [3]). Likewise, this happens where their deformable images are of certain applications [8], [9] between differential operators in a holomorphic framework that can be determined establishing actions on said elements (that are ramifications). The actions from loop groups obtained in the construction process of cycles of the space-time [3] are the ramifications.

II. SPECTRUM OF THE COMMUTATIVE RINGS: THE IMAGE OF A SPIN MANIFOLD

Let X, be a scheme as studied by Milnor [4], more specifically, the scheme given from the derived categories D_X, whose sheaves I, are coherent sheaves of ideals on X, then the transformation that we define is the morphism □: $X^{\sim}$ □X, such that □$_{□1}IO_{X^{\sim}}$, is an invertible sheaf. Here $O_{X^{\sim}}$, is the structure of the sheaf of $X^{\sim}$. For this way, morphisms from schemes to affine schemes must be understood in the ring homomorphisms context by the following adjoin contravariant pair (X, A): For every scheme X, and every commutative ring A, we have the natural equivalence [1], [9]:

$$\mathrm{Hom}_{\mathrm{Schemes}}(X, \mathrm{Spec}(A)) \, □ \, \mathrm{Hom}_{\mathrm{CRing}}(A, O_X(X)), \qquad (1)$$

due to that A, is an initial object in the rings category before the functor action Spec, in the schemes category, and their

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final object is Spec(A) , which means the spectrum of commutative rings category. The character Spec is the functor
"spectrum".

Likewise, we consider the Axion particle which must be a final element or object in the space Spec(A). **Definition 1.** A spin manifold in the image of Spec(A)of the space-time, is an axion.

An axion is not a spin manifold; however, this is the result of a transformation "blow up" in the space-time of a spin manifold in QFT-frame. Likewise, the axion can be determined and defined as an image of a spin manifold under transformation rules in the Universe modeled as a complex Riemannian manifold (of Kahlerian type) whose dimension is 2n [7]-[9]. Thus, in moduli problems, we can obtain equivalences in duality of the objects in A, andSpec(A), and their images in a complex Riemannian manifold.

If we consider as model of the space-time the complex Riemannian manifoldM :□ ` $_H(G, C)$, the equivalences can be defined in $D$□branes (that is to say $D$□modules), and strings level through of the equivalences established in Theorem 7. 1 [5] , having that, under certain duality, ` $_H(G, C)$, is composed for objects of a derived category of D□modules on $B_G(C)$, (here $B_G(C)$, as category is a category of Hecke eigensheaves) where this last is a category of the G□ equivariant D□modules where C, is a complex whose scheme Y (that are orbifolds of a CY-manifold, which is a Calabi-Yau manifold. This is a spin manifold if establishes equivalences between
D□modules of $B_G(C)$,and D□branes in the moduli space
` $_H(G, C)$) is the spectrum of G□ equivariants D□modules (C,is a complex of certain special sheaf of holomorphic Gbundles (of certain eigen-sheaf of Hecke)). Then can be obtained a spectrum of the space-time.

In the study of the algebraic geometry and the theory of complex analysis on manifolds, the coherent sheaves are a specific class of sheaves having the particularity of establishing properties of differential operators linked to the geometrical properties of the underlying space where these are defined. The geometrical information codification of coherent sheaves is realized with reference to a sheaf of rings.

III. Topological Quantum Diffeomorphisms in Field Theory

Using the model in Fig. 1, [1], let the corresponding noncompact Lagrangian submanifolds L, as homotopy of L_o,where L_o,is the Lagrangian submanifold before of the field action given by one particle, to know [4]:

$$HW^*(L_o,L_o) \,\square\, H(CW^*(L_o,L_o)), \tag{2}$$

We consider the Fukaya conjecture taking as cotangent fiber L□T_x*,to the A□ □structure on complex $CW^*(L_o,L_1)$, which is isomorphic to the algebra dg□ structure on the deformed category C□□(□$_x$),of the loop space (Z, x), with Z,a differentiable manifold which has a normalized geodesic flow accord to their Poincaré section [2], [6].

Remark 1. A free loop space or loops space□X □$C(S^1, X)$, of the topological space X,is the space conformed for loops from the unit circle S^1, to X, endowed with compact-open topology.

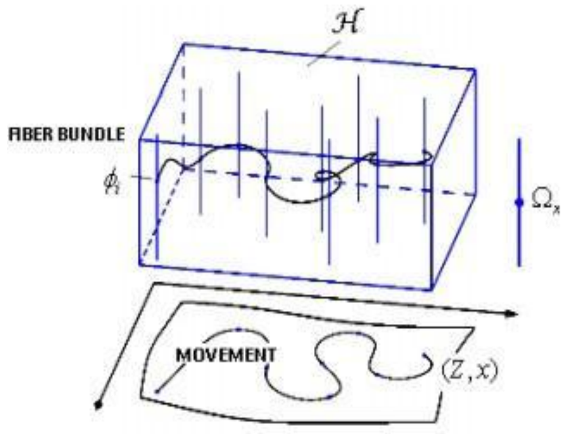

Fig. 1 Correspondences between states (L,x), of the Hamiltonian space H ,and points (Z,x),of the loop space

Due to the TFT, the flow is a geodesic flow, which is conformed for trajectories defined by the variation principle. The flow is created from the micro-states that have been measured by gauge fields [1] in a Hamiltonian space as state space [8], [9].

From the *conjecture 1*, given in [6] the movement is spread ramification from the energy-vacuum mechanism through paths of particle actions given by [7]

$$\square_{O_c}(\square) \,\square\, \square_M O_c(\square(x))d\square(x), \tag{3}$$

The Bulnes's operator O_c[7], involves a connection

□$_t(X_p)$,□p□M, X_p □$T_p(M)$ of the tangent bundle of the space of trajectories□(□) □R^3□ I_t, such that the map

$$X \,\square\square_X(1), \tag{4}$$

Indeed, is the affine connection describes as:

$$□_t(X_p) □□_{□'(t)}X □ \exp_p(tX),$$

which is a diffeomorphism from $U_o □T_p(M)$ to $U_p,□p□M$, being U_o,U_p, open neighborhoods.

Remark 2. $□_{X^t}$ is the flow of geodesics □, such that $□X$, is a
Hamiltonian vector field of H, which satisfies (4).

What is the value of this diffeomorphism in the quantum context? We consider the same topology. Is this also an image of the other diffeomorphism?

Could an extended topology be necessary and also induced [1], [2], [8]? Thus, the paths to the quantum image of the space-time M (considering the field actions in (3)), take the form to the energy states collection [2], [6]:

$$□_{O_x}(□) □ \int_{X(M)} O_c(□(x))d□(x), \qquad (5)$$

preserving the same symplectic structure [1], [9].

A correspondence between wrapped Floer cohomology and the ordinary Floer cohomology can be established in [1], [9], given by $H^*(L)$, [8]. Here, the action functional (3) has the same value with (5).

The wrapped Floer cohomology that wraps the flow of geodesics $x: [0, 1] □M$, of the Hamiltonian H, generates the associated Floer cochain complex, which goes from L_o,to L_1; having that

$$CF^*(L_o,L_1;H) □ CF^*(□^1_X(L_o),L_1), \qquad (6)$$

where this must show a little variation to create in the infinity, the wrapping such that given two $dg□$modules E_L, and E_Q,

(this is a $dg□$module on an $dg□$algebra of Cěch cochains, where these co-chains are obtained in duality. This, in other re-interpretation, could be a complex sheaf that can be generalized in complex cohomology to one Hochschild cohomology type) as elements of a $dg□$category defined by $M □ mod(C)$, where C, is the Cěch complex defined by (where $d □|□_1$) [9], [10]:

$$C□ \prod_I □(U_I)[□d], \qquad (7)$$

Theorem 1. (Abbondandolo-Schwartz) [9]. Let $M □ T^*Z$,be the cotangent bundle of a closed oriented manifold [1], [9], and take two cotangent fibers $L_o □T_{x□_o}Z$ (that is to say, the obtained from O_c) and $L_1 □T_{x□_1}Z$.Then

$$HW^*(L_o,L_1) □H_{□□}(L_{x_o,x_1}), \qquad (8)$$

is the (negatively graded) homology of the space of paths in Z, going from x_o, to x_1.

Proof. [9].

Conjecture 1. [8], [10]. Let $M □T^*Z$,be a cotangent fibre. Then the $dg□$module E_L (of Lagrangian submanifold L) is isomorphic to $E_L$in $H(M)$. Moreover, if it is simply connected, G, gives rise to a quasi-isomorphism

$$C_{□□}(□_x) □CW^*(L,L) □ \hom_m(E_L,E_L), \qquad (9)$$

Remark 3. G is the $A_□$ □functor: $G: W(M) □M□ \mathrm{mod}(C)$, it is resulted from associates to any exact Lagrangian submanifold a $dg□$ module E_L, over C. and which finally, is Legendrian at infinity. Here, an $dg□$algebra over our coefficients fieldK, is the defined with an augmentation $□:C □K$, whose kernel is denoted by I. Then, it is possible to equip the free tensor coalgebra $T(I[1])$,.with a differential where this co-algebra can be dual to an $dg□$algebra $B □ T(I[1])□$.

Theorem 1. (F. Bulnes, M. Ramírez, L. Ramírez, O, Ramírez). The wrapping category (spectrum) is characterized
by the fields (in physics) related by the diffeomorphism $C_{-□}(□_x) □W(H)$, whose paths space going from□(x), to □(x), as in (8). Then, the field ramification is the connection to the operator $O_c: TM□T^*M$.

Proof. [1], [6].

IV. Results

From the proposition 3.1, the important statement of that brings a wrapping homology (as a product of homology from the deformation theory) that gives support and fundament to the topological diffeomorphisms in QFT, to the space and time, gives place to their field ramification (the connection of the loop space $C_{□□}(□_x)$ [2], [9], [10]), which is a connection obtained under the legitimate scheme [1], [2], [11]:

$$O_c(□)□H(\mathrm{mod}\, f(C_{□*}(□Z)))□□□^R□^{□_1}H(M)□□□ \quad C$$
$$□ \qquad □\mathrm{embb} \qquad □_G \qquad (10)$$
$$C_{□*}(□_x) \quad □□□^{\mathrm{Diff}}□ \; W(H) \; □□□M$$

where the distinguished diffeomorphism $C_{-□}(□_x) □W(H)$,is demonstrated in the proposition [1], [9]. Field ramifications are considered the descendent mappings that under certain transformations in geometrical stacks can be topological spaces [2], [6] created by the corresponding flows of minimal geodesics, $D□$ branes, or strings of many types [1]. Thus, considering the functoriality scheme of rings established in (1) and having the connection scheme to the space-time M, ramified field [1], [9] we can enounce the following generalizing of (11) given by the Theorem.

Theorem 2. (F. Bulnes) [1], [12], [13]. If we consider the category $M_{K_F}(\hat{g}, Y)$,then a scheme of their spectrum $W(H)$, where Y, is a Calabi-Yau manifold comes given as:

$$\mathrm{Hom}_{\hat{g}}(X, V_{criticalDef}) \square \mathrm{Hom}_{Loc_{LG}}(V_{critical}, M_{K_F}(\hat{g}, Y)), \quad (11)$$

Proof. [1], [2], [14].

The spectrum in (11) is a Verma module of critical level. In the case to consider a homotopy category $H_0(C)$, of a differential graded category C, then the isomorphism given in (11) has in some time, the same objects as C, but their morphisms are defined by the identity:

$$\mathrm{Hom}_{H_0(C)}(L_0, L) \square H_0 \mathrm{Hom}_C(L_0, L), \quad (12)$$

which is doing re-counting to image of spectrum over all space-time M , considering this as the Hitchin space [15] $`_H(G, C)$, to different stratus of dimensional spaces, we can derive an identity of the moduli spaces considering their different geometrical and physical stacks [16].

We demonstrate the commutative scheme given in (10), using the fact that

$$\mathrm{embb}\,(R^{\square 1}(O_c(\square)))(\square) \square \mathrm{Diff}\,(\square)(x(t)), \quad (13)$$

Using arguments of homotopy [17], and the natural chain mappings or diffeomorphism of the type (in homological algebra, the homotopy category of chain complexes in an additive category C, is a framework for working with chain homotopies and homotopy equivalences [17]. This is an intermediate between the category of chain complexes of C, and the derived category of C, when C, is Abelian; unlike the former it is a triangulated category, and unlike the latter its formation does not require that C, is Abelian):

$$\square : C(L_0, L_1) \square WH(L_0, L_1) \square WH(L), \quad (14)$$

where $WH(L) \square W(H)$,we obtain images $C \square WH$,which complete the diagram (10).

In the general case, that is to say, in all space M , will be necessary use the natural chain mapping

$$\square : CW(F) \square C_{\square *}(\square(M)), \quad (15)$$

here $F \square T_q^* M \square T^* M$.

V. Conclusion

The spectrum of the deformed derived category given for can be viewed as an aspect of one version of the Floer cohomology type and their Lagrangians (Lagrangian submanifolds) that are the object (points of the super-manifold defined by the Hitchin moduli space $`_H(G, C)$) in the other class of the homomorphism $L_0 \square L_1$, that includes the ring structure [2] to preserve commutativity in the diagram (10). This is the principal conclusion derived of the Fukaya conjecture, and schematized through (10), which is commutative diagram to geometrical Langlands ramification of the operator $O_c : TM \square T^* M$. Likewise, this ramification is given by a connection deduced from the mentioned operator whose states are in the modules of the category $WH(L) \square W(H)$,where each Lagrangian submanifold is a kernel to the Floer homology complex, that is to say, $WH(L) \square 0$.Then, a more general sense, that is to say, to category of Lagrangiands $F \square T_q^* M \square T^* M$,we have (15).

Then in wrapped Floer cohomology we have a field equation as for example $W^{\square} \square 0$,(Weyl equation) or the Dirac equation

$D \square \square 0$.

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