JCE
EST
1998
Journal of Computer Engineering
ISSN 10003428

Original Research Article
Vol 9, Issue 9, 2020 | ISSN: 1000-3428

Model-Based Predictive Control for Non-Integer Order Systems via LMI Optimization

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***Abstract*—**In this paper, the problem of robust model predictive control (MPC) for discrete-time linear systems in linear fractional transformation form with structured uncertainty and norm-bounded disturbance is investigated. The problem of minimization of the cost function for MPC design is converted to minimization of the worst case of the cost function. Then, this problem is reduced to minimization of an upper bound of the cost function subject to a terminal inequality satisfying the l_2-norm of the closed loop system. The characteristic of the linear fractional transformation system is taken into account, and by using some mathematical tools, the robust predictive controller design problem is turned into a linear matrix inequality minimization problem. Afterwards, a formulation which includes an integrator to improve the performance of the proposed robust model predictive controller in steady state condition is studied. The validity of the approaches is illustrated through a robust control benchmark problem.

I. INTRODUCTION

MPC technique was firstly developed for oil refining applications in the 1970s. During past decades, the use of MPC increased in several other fields, such as the chemistry, aerospace, and food industries [1]. Novel applications have included, for example, the control of oxygen excess ratio in fuel cells [2], management of battery/supercapacitor storage systems in hybrid electric vehicles [3], exhaust emission regulation in turbocharged diesel engines [4], and load voltage control of four-leg inverters [5].

One of the most important reasons for the wide acceptance of MPC in industrial applications is the possibility of handling constraints on manipulated and controlled variables [6], [7]. Nominal stability and constraint satisfaction guarantees can be obtained with the adequate formulation of the optimization problem to be solved [8], [9]. However, such properties may be lost in the presence of a mismatch between the internal model of the controller and the actual dynamics of the plant, resulting from modeling simplifications, parametric uncertainties, or disturbances.

In this context, some research studies have been conducted to develop robust MPC (RMPC) formulations. Various RMPC theories are developed by the researchers during past decades.

They attempt to deal explicitly with plant model uncertainty which was the most important disadvantages and the inability of the previous MPCs [10]-[14]. Early propositions involved uncertainties expressed in the form of bounds on the impulse response of finite impulse response (FIR) models [15], [16]. A more elaborate approach introduced by Kothare et al. [10] allowed for the use of more general uncertainty structures, either in polytopic or structured feedback forms. The resulting optimization problem could be cast into a semidefinite programming format, with constraints in the form of linear matrix inequalities (LMIs). The goal in previous design was a state feedback control law which minimizes a worst-case infinite horizon objective function, subject to constraints on the control input and plant output [10]. There are few RMPC methods presented in the literature that consider both model uncertainty and disturbances as it is crucial in some physical models. This can be attributed to the fact that there is always a trade-off when both model uncertainty and disturbances are considered; thus, the researchers consider only one of them as the sources of these two inconveniences are different [14]. Therefore, a RMPC design with respect to model uncertainty and disturbances has yet to be realized. The goal of this paper is to develop RMPC theory for a class of linear systems using state-of-the-art advanced control, MPC and robust control strategies. The RMPC is to provide better performance as it should be robust against model uncertainty and induced disturbances.

In this paper, a RMPC design methodology for a class of discrete-time linear systems is investigated. The method presented in this paper is developed based on a LMI design procedure for the online state feedback control. The main contribution is the accomplishment of prescribed disturbance attenuation in a systematic way by incorporating the wellknown robustness guarantees. To this end, a quadratic Lyapunov function to guarantee the stability of the close-loop linear system is presented. The problem of minimization of the cost function for MPC design is altered to the minimization of the worst case of

How to cite: Dr. Ramesh Nair, Dr. Astrid Jensen (2020). Model-Based Predictive Control for Non-Integer Order Systems via LMI Optimization. Journal of Computer Engineering, 9(9), 59–69. DOI: https://doi.org/10.5281/zenodo.19342117

the cost function. Then, this problem is converted to finding the upper bound of the cost function subject to the $\square_2$-norm robust constraint. Due to the possibility to recast the robust problems in LMI format, the problem is turned to be a LMI minimization problem to be solved. Furthermore, in order to improve the steady state response, integrator control approach is considered to be added to the formulation of RMPC. The controller design procedure is illustrated through a benchmark problem of robust control widely used in the literature to depict the validity and performance of the proposed RMPC scheme.

The rest of the paper is organized as follows. In Section II, the linear system in fractional transformation format is described, and then, the problem of RMPC is illustrated. In Section III, the formulation of integrator RMPC is presented. The simulation results of the two approaches in this paper are brought is Section IV to make a comparison between the two strategies. This paper is concluded in Section V.

II. ROBUST MPC PROBLEM (RMPC)

A. Linear System in Linear Fractional Transformation Format

Different kinds of models are available in the literature for different dedications including control and robust study purposes [17], [18]. A common paradigm for robust control is a linear model with uncertainty appearing in the feedback. Consider the following discrete-time linear system in linear fractional transformation (LFT) form,

$$\begin{aligned} & x(k\square\ \square 1)\ Ax\ k(\ )\square Bp\ k(\ )\square B\ u\ k_u(\ )\square B\ w\ k_w(\ ) \\ & q\ k(\ )\ \square Cx\ k(\ )\square Dp\ k(\ )\square D\ u\ k_u(\ ) \\ & p\ k(\ )\ \square\ \square(\ q\ k)(\ ) \\ & y\ k(\ )\ \square C\ x\ k_y(\ ) \end{aligned} \tag{1}$$

where $\square$ is a block diagonal introduced here,

$$\begin{matrix} \square\square_1 & & \square \\ \square & & \square \\ & \square_1 & \\ \square\ \square\ \square & & \square \\ \square & & \square\ \square \\ \square\square & & \square_r\,\square\square \end{matrix} \tag{2}$$

$x\ k(\ )\square\square^n$, $u\ k(\ )\square\square^m$, $w\ k(\ )\square\square^r$, and $y\ k(\ )\square\square^z$ are the state, controlled input, exogenous input, and controlled output, respectively. Fig. 1 illustrates the common LFT format. $w\ k(\ )$ here is the norm bounded disturbance, so,

$$\|\ w\ k(\ )\ \|_2 \square W_{\max} \tag{3}$$

where, $W_{\max}$ is a known value.

In this framework, $\square$ can represent a number of factors including nonlinearities, parameters and also dynamics that are unknown or uncertain. Moreover, $\square$ illustrates a convolution operator with the operator norm induced by the ℓ_2-norm less than 1, i.e.

$$\square_{j\square 0}^{k}\ p\ j_i(\ )^T p\ j_i(\ )\ \square\ \square_{j\square 0}^{k}\ q\ j_i(\ )^T q\ j_i(\ ),i\ \square 1,\ldots,\ ,r\ \square\ \square k\quad 0 \tag{4}$$

B. Model Predictive Control (MPC)

MPC is an open-loop control design method. In this method, plant measurements and a model of the process are used in order to predict the future outputs of the system. Using the aforementioned predictions and minimizing a cost function over prediction horizon introduced in (5), m control moves $u\ k(\ \square i\ k|)$, $i\ \square\ 0,1,2,\ldots,\ m\ \square 1$are obtained.

$$\min_{u\ k\ i\ k(\ \square\ |\ ),i\square 0,1,\ldots,m\square 1} J(\ )^k \tag{5}$$

In MPC framework, only the first computed input $u\ k\ k(\ |\ )$ is considered for implementation. For the next step, the optimization problem (5) is solved again with respect to receding prediction horizon and control horizon. This procedure is continued, and new measurements are used at each sample time until the whole control input is obtained. It is assumed that, after the time $k\ m\square\ \square 1$, there is no more control action. There is an option to add control signal and states constraints to the optimization problem (5) to more properly control the system. This is the main advantages of the MPC [19].

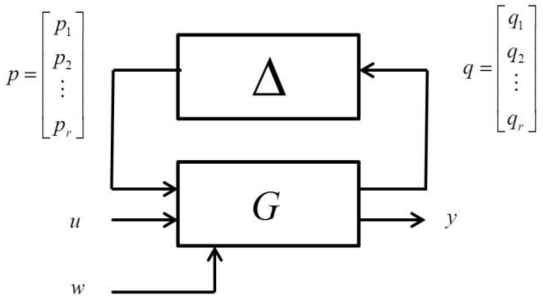


Fig. 1 Graphical representation of structured uncertainty with disturbance

C. Linear Matrix Inequality

Two well-known lemmas are brought in this section. In order to construct an optimization problem based on LMIs, these two lemmas play an important role. For more details of LMIs and their solvers for optimization problems, one can refer to [20], [21].

Lemma 1. (Schur Complement): Convex quadratic inequalities can be converted to LMI using Schur Complement. Consider that symmetric matrices $Q\,x(\,)$, $R\,x(\,)$, and $S\,x(\,)$ depend affinely on x. Then, the following linear matrix inequality and the equation inequalities are equivalent [22].

$$\square Q\,x(\,)\ S\,x(\,)\square$$
$$\square\square S\,x(\,)_T R\,x(\,)\square\square\square\ 0$$
$$R\,x(\,) \square 0,\ Q\,x(\,) \square S\,s\,R\,x(\,)\,(\,)^{\square 1} S\,x(\,)^T \square 0$$
$$Q\,x(\,) \square 0,\ R\,x(\,) \square S\,s\,Q\,x(\,)^T \quad (\,)^{\square 1} S\,x(\,) \square 0$$

(6)

Lemma 2. (S-procedure [22]): Consider that $Q_i \square \square^{n\,n\square}, i \square 1,2,\ldots,q$ are symmetric matrices. The conditions on Q_i,

$$\cdot\cdot\overset{T}{Q}_0\cdot 0, \quad \cdot\cdot\cdot\ 0 \quad s\ t.,$$
$$\cdot\cdot\overset{T}{Q}_i\cdot 0, \quad i\cdot 1,2,\ldots,q \qquad (7)$$

hold if there exists $\square_i \square 0,\ i \square 1,2,\ldots,q$ such that,

$$Q_0 \square \overset{q}{\underset{i\square 1}{\square}} \square_i Q_i \square 0 \qquad (8)$$

D. RMPC

Considering a linear uncertain system described by (1), for RMPC design, the minimization problem at each sampling time k, of the nominal performance objective in MPC design is replaced by the minimization of the worst case,

$$\min_u \max_{w,\square} J_\square(\,)k \qquad (9)$$

where

$$J_\square(\,)k \square \overset{\square}{\underset{j\square 0}{\square}} [\,(x\,k \square j\,k|)^T Q\,x\,k_1(\square j\,k|\,) \qquad (10)$$
$$\square\ \square u\,k(j\,k|)^T R u\,k(\square j\,k|)\ \square\square^2 w\,k(\square j)^T R\,w\,k_w(\square j)]$$

Here, we consider an online state vector feedback controller, so,

$$u\,k(\,)\square K\,k\,x\,k(\,)\,(\,) \qquad (11)$$

In the presence of disturbance, the task is to minimize the ℓ_2 gain between the disturbance input $w\,k(\,)$ and the output $y\,k(\,)$. In the linear case, the $\square_2$ gain is called $\square_\square$ norm.

The problem of $\square_\square$ filter design is to determine matrix *K(k)* such that:

$$\lim_{k\square\square} y\,k(\,) \square 0 \ for\ w\,k(\,) \square 0$$
$$\|y\,k(\,)\|_2 \square\square \|R_w^{0.5} w\,k(\,)\|_2 \ for\ w\,k(\,) \square 0 \qquad (12)$$

Consider a quadratic Lyapunov function such that,

$$V \square x\,Px^T,\ P \square 0\ \&\ V(0) \square 0 \qquad (13)$$

Suppose that V satisfies the following inequality,

$$V\,x\,k((\square\square i\,1|\,k)) \square V\,x\,k((\square i\,k|\quad)) \square \qquad (14)$$
$$\square((x\,k \square j\,k|)^T Q\,x\,k_1(\square j\,k|) \square u\,k(\square j\,k|)^T R u\,k_1(\square j\,k|\quad)$$
$$\square\square^2 w\,k(\square j)^T R\,w\,k_w(\square j))$$

For robust performance objective function to be finite, we assume that, $x(\square,k) \square 0$ and so, $V\,x((\square,\,))k \square 0$.

Summing the last inequality from $i \square 0$ to $i \square\square$, we have,

$$\square V\,x\,k\,k((\quad|)) \square\square J_\square \qquad (15)$$

Therefore,

max*w*,□ *J*□( )*k V x k k*□ ((
|)) (16)

The goal is that at each sampling time *k*, a constant state vector feedback law created to minimize the upper bound of *V x k k*((|)) . In other words, the problem of □□ filtering design in the presence of disturbance is reduced to find a Lyapunov function *V x k k*((|)) > 0 such that,

v □□ □*Vy k y k*( )*T*( ) □□²*w k R w k*( )*T w*( ) □ 0
(17)

is negative definite, where

□ *V*□ *V* (*x k*(□ □*i* 1 | *k*)) □ *V* (*x k*(□ *i* | *k*))
(18)

As it is standard in MPC, only the first computed input *u k k*(|) □*K k k x k k*(|) (|) is implemented. At the next sampling time, the state *x*(*k k*|) is measured, and the optimization is repeated to recompute gains *K k*( ) . The following theorem gives us conditions for the existence of the appropriate *P*□ 0satisfying (13) and the corresponding state feedback matrix *K k*( ) .

Theorem 1. For given control parameters, □, *Q R*₁, ₁ and *R*w , the RMPC design problem for a system with structured uncertainty and norm-bounded disturbance is achieved if there exists *Q*□0, □□0, *P* □□*Q*□¹, *Y KQ*□ , □ □ □□ □ □¹ □ 0 , □ □□*i*□ □ *i*□¹ □ 0, *i* □ 1,...,*r* that solve the following convex optimization problem:

m in ·
Q Y, ,· , · · (19)

Subject to,

··1 *W*max2 *x*T ·
· ·2 ·
. * ··· 0 ·· 0
. * · *Q*·· (20)
·

··*Q* *Y R*T 0.5 *QQ*10.5 *QC*clT *QA*clT 0 0 ·
· ·
. * ··*I* 0 0 0 0 0 ·
. * * ··*I* 0 0 0 0 ·
· * * ··· 0 ·
. * * * * ··*D* 0 · · 0
. * * · ··*B*p ·*B*w ·
· * *Q* ·
. * * * * ··· 0 ·
. * * * *
· * ··² *R*w ·. (21)

where, *A A BK*cl · · u and *C C DK*cl · · u .

If the problem is feasible, the control action *u k*( ) is obtained as, *u k*( ) □ *K k x k*( ) ( ), where,

K( )*k Y k Q k*□ ( ) □¹( ) (22)

Proof: Using *u k*( ) □*Kx k*( ) , and considering a quadratic

Lyapunov function *V x Px*□ *T*, *P*□0 and supposing that *x*(*k k*|) □ *x* for simplification, the variation of the aforementioned Lyapunov function is given by,

□ □*V*(*Ax* □ □*BpB Kx*u□ *B w*w)*T P Ax*(□ *Bp*

□ *B Kx*u □ *B w*w) □ *x Px*T (23)

Based on the Lyapunov stability theory, the convergence of the state vector *x(k)* to zero is guaranteed if the terminal inequality (17) is verified, which holds if

v□□ □*T M* □ 0 (24)

where

□*T* □[*x*T*p w*T *T*] (25)

and,

□(*A BK PA BK P Q KRK A BK PB A BK PB*□ u) (*T*□ u)□ □ ₁□ *T*₁
(□ u)*T* (□ u)*T* w□

M□□ □ *BPB*T *BPB*T w□ □0
(26)
□ □
□□ □ □ *BPB*w*T*w □□

Moreover, for the uncertain parameters introduced by block diagonal □ , we have (this also can be seen in [10])

p kj(□ i k|)Tp kj(□ i k|) □ ((C □ D K x ku) (□ i k|) □

Dp kj(□ i k|)) ((TC □ D K x ku) (□ i k|) □ Dp kj(□ i k|)),

j □ 1, 2,..., r

(27)

Therefore, gathering all the equations j □ 1,2,...,r , also from the results of lemma 2, we obtain

· ·x T ·· ·CclT Ccl · ·CclT D 0·· ·x

· ·p · · · ·(I D DT) 0 ·· · p · 0

· · · · · ·

·· ·w ·· ·· · 0··· ·· ·w

(28)

with,

··1I ·

· ·

□I

□ □ □ 2 □ □ 0 (29)

□ □ □

□□ □rI □□

It is easy to see that both of the previous equations are satisfied if there exist □□ □1□, 2□,..., r□ □ 0 such that,

□A PAclTcl□ P □ Q1□ K R KT1□ CclT□□Ccl

M □ □ □

□

□□ □

(31)

A PBclT · CclT A PBclT w ·

··D ·

B PBT w

B PBT · ··(I · D DT) · · 0

□ B PBwTw □□2Rw □□

□□1□I □

□ □

□ □□I □

□□ □ 2 □ 0

□ □

□ □ □

□□ □r□I□□

Substituting P Q□□ □1 with Q □ 0 , after some straightforward manipulations, we see that this is equivalent to the existence of Q □ 0 , Y □ KQ , □ □□ 0 such that (32) holds.

In (32), □ □ □□ □□1□ 0 and □ □□□ □□1 □ 0 . On the other hand, assuming that lim ( )x k □ 0 (justified by the asymptotic x□□ stability of x( )k , we have,

□ □ □V x k · Px k( ) ( ) T

(33) l k□

Taking the sum of both sides of the terminal inequality (17) from k=1 to k→ ∞, and some simple manipulations, we reach (20). The proof is complete.

··Q Y RT0.5 QQ10.5 QCclT QAclT 0 0 ·

· * ··I 0 0 0 0 ·

· * * ··I 0 0 0 0 ·

· * * ··· 0 ··D 0 ·

· * * * * ·Q ··Bp 0 · · 0

· * * * * * ···

· * * * * * ·Bw ·

· * 0 · (32)

· * ··2 Rw ··

Remark 1. It is worth saying that the proposed method for RMPC guarantees both the performance and robustness in presence of disturbances and model uncertainty.

Remark 2. It is mentioned in theorem 1 that for a given □, we search for an upper bound for robust performance. Instead, this minimization can be solved with respect to □, reaching to the best design for disturbance rejection or performance.

III. INTEGRATOR RMPC FORMULATION

The new formulation for RMPC is developed to improve the steady state response of the proposed RMPC in the previous section.

Consider the system introduced by (1). The problem of integrator RMPC is constructed by adding a new state x_I to

(1). Then, (1) is altered to,

□ x k(□1) □ □ A 0□ □ x k( ) □□Bu □

□ xI (k □1)□□ □ □□□C y0□□ □□ xI ( )k □□ □ □□ 0 □□ u k( ) (34)

□

□Bw □ □B p □ □ □0

□ □ □ w k( ) □ □□ p k( ) □ □ □ r

□ 0 □ □ 0 □ □ □1

where,

□ □x- □ A

0□ x-□□ □xI ,

A□□□□Cy

0□□,

□ □

_ □ □Bu − □Bw □ − □Bp □

Bu □□ □0 , Bw □□□ 0 □□, Bp □□□ 0

□□ (35)

□ □

Thus, the linear fractional transformation of the new formulation is introduced in the following,

x(k □ □1) A x( )k □ B p kp ( ) □ B u ku ( ) □ B w

kw ( ) q( )k □C x( )k □ D p( )k □ D u ku ( )

p( )k □ □(q)( )k

y( )k □C y x( )k (36)

By considering the aforementioned formulation, the new integrator RMPC formulation is driven similarly to the RMPC scheme introduced in Section II.

There is a trade-off between the performance and disturbance rejection in the control design. In integrator RMPC formulation, the accuracy of the performance of the system's steady-state response is increased significantly although the overshoot of the system increased. One possible way to eliminate this weakness of the integrator RMPC problem is to switch between integrator RMPC and RMPC introduced in this paper. To do so, when the states of the system reach to a predefined bound around the origin, the switching control is initiated between two strategies. This strategy significantly improves the performance of the system. This switching strategy is illustrated through an example in the next section.

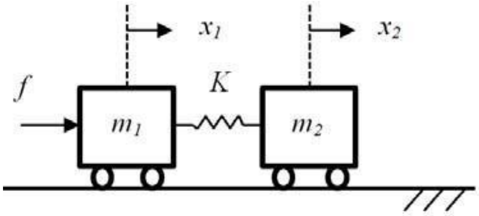


Fig. 2 A tow-mass-spring system

IV. SIMULATION RESULTS

In this section, the results of the proposed strategies in previous sections are illustrated via a benchmark example for robust control design. Fig. 2 presents a two-mass–spring system employed in this paper as an example. This example is acquired from [10]. Using Euler first-order approximation with a sampling time of 0.1 s, the discrete-time model of this system is achieved. The model in terms of exogenous variables is described in (37) and (38). x_1 and x_2 are the positions of the active cart and passive cart, and x_3 and x_4 are their velocities, respectively. m_1 and m_2 are the masses of the two bodies, and K is the spring coefficient.

$$A = \begin{bmatrix} 1 & 0 & 0.1 & 0 \\ 0 & 1 & 0 & 0.1 \\ -0.1K_n/m_1 & 0.1K_n/m_1 & 1 & 0 \\ 0.1K_n/m_2 & -0.1K_n/m_2 & 0 & 1 \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ 0 \\ 0.1/m_1 \\ 0 \end{bmatrix}, \quad B_p = \begin{bmatrix} 0 \\ 0 \\ -0.1 \\ 0.1 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ 0 \end{bmatrix} \quad (37)$$

$$C = \begin{bmatrix} 0.475 & 0.475 & 0 & 0 \end{bmatrix}, \quad C_y = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \quad (38)$$

In (37), we assume that $m_1 \square m_2 \square 1kg$ for masses and K is the uncertain constant such that, $K_{\max} \square 10$, $K_{\min} \square 0.5$ as the maximum and minimum value of the uncertain spring. The uncertainty is modeled similar to (1), thus,

$$K_n \square \frac{K_{\max} \square K_{\min}}{2}, \quad K_{dev} \square \frac{K_{\max} \square K_{\min}}{2}, \quad \square \square \frac{K K \square_n}{K_{dev}} \quad (39)$$

For the objective function introduced in this paper, we assume that, $Q_1 \square diag(1,1,1,1)$, $R \square R_w \square 1, \square \square 5, W_{\max} \square 0.5$.

Three methods including that of Kothare et al. [10], RMPC, and Integrator RMPC introduced in this paper are compared. The target is to bring the position of the carts to $x_d = [1,1]$. Therefore, shifted states, shifted inputs, and shifted outputs are defined as $\square x = -x\, x_d$, $u\square = -u\, u_d$, $\square y = -y\, y_d$, respectively.

Then, the origin of theorem 1 is moved to the steady state value. In this example, the initial conditions are assumed such that all the states of the system start from the origin. Figs. 3-5 show the simulation results for the aforementioned methods. Figs. 3 and 4 depict the position and the velocity of the carts, respectively. In this figure, the RMPC introduced here in comparison to the method of Kothare et al. has a faster response. Furthermore, the integrator RMPC method increased the overshoot of the system. Both of the proposed methods in this paper are able to steer and also keep the states to the reference in the presence of uncertain parameter and induced disturbance. Fig. 5 illustrates the optimal value of □, the answer of the minimization problem (19) in each step k. As it is clear in this figure, both formulations introduced in this paper reach to a smaller optimal value for □. Moreover, Fig. 5 shows the norm of feedback gains, *K(k)*. It can be shown from this figure that the proposed formulations in this paper reach to a bigger norm of K in comparison to the method in [10].

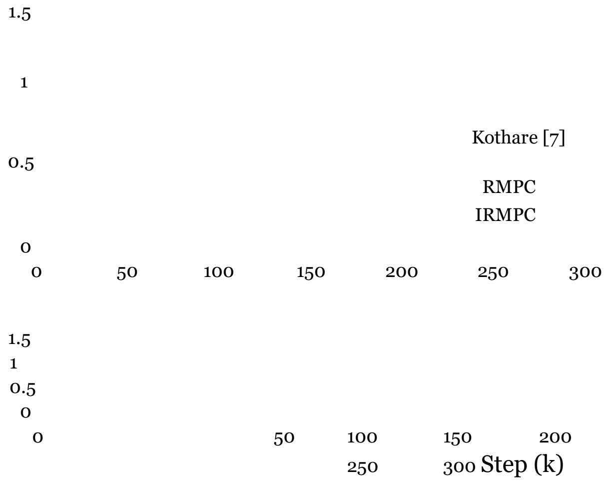


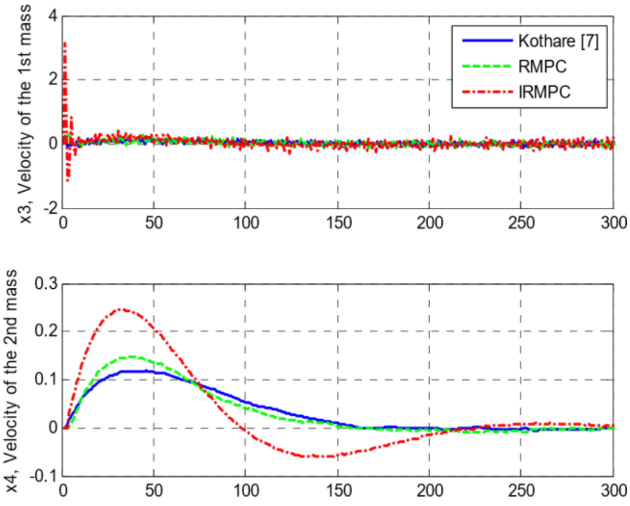
Fig. 3 Position of the carts

Another study has been done in order to illustrate the performance of the controllers when a switching control law is considered for control system. To do so, the RMPC illustrated in this paper is activated at first. Then, as the position of the 1st cart is reached to the 10% of its steady state value, the integrator RMPC triggered to steer the states of the system to the reference value. Simulation results for switched strategy are shown in Figs. 6-8. Fig. 6 illustrates that the switching control strategy in comparison to the method of Kothare et al. improves the performance of the system such that the switching strategy has the ability to decrease the settling time without overshoot. Fig. 7 shows the velocity of the carts for the switching strategy between the RMPC and integrator RMPC and in comparison to the method in [10]. This figure shows the ability of the proposed switching strategy to steer the velocities to zero such that it improves the performance of the system compared to the method in [10]. Fig. 8 depicts the norm of the feedback gains and the optimal value of □ for switching control strategy and the method in [10]. Although the feedback gains in switching controller are bigger than the method of Kothare et al., the optimal value of □ is less than the method in [10].

V. CONCLUSION

In this paper, a RMPC design technique using LMIs for time invariant discrete-time linear systems is developed.

The uncertain system was represented in linear fractional transformation format. The controller development is based on state feedback control and Lyapunov stability theorem. The online optimization problem to achieve the feedback gains contains the solution of a LMI minimization problem. The LMI minimization problem is a convex optimization problem. Hence, the resulting state feedback control law minimizes an upper bound on the robust objective function. Besides, an integrator is added to the formulation in order to increase the accuracy of the responses in steady state condition. This paper shows that we have been able to handle uncertainty and disturbance in RMPC design approach with high accuracy for the reference tracking. A benchmark example illustrated capability of the proposed methods.



Step (k)

Fig. 4 Velocity of the carts

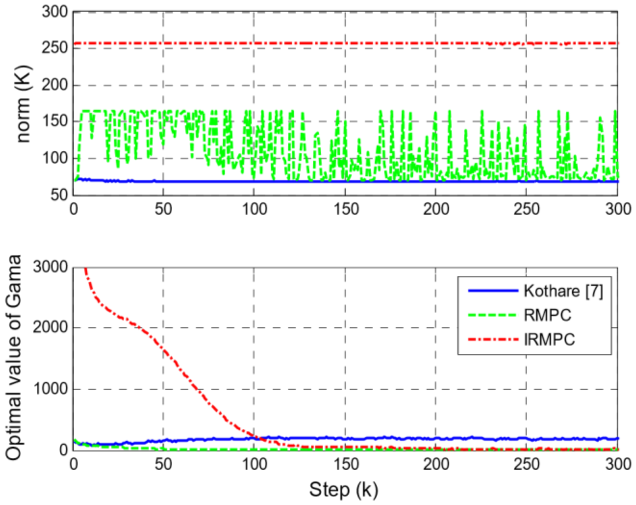


Fig. 5 Norm of the feedback gains and the optimal value of the Gama

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