

Numerical Modeling of Pulse Signals in Collisional Cold Plasma

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Abstract—The influence of inhomogeneities of plasma and statistical characteristics on the propagation of signal is very actual in wireless communication systems. While propagating in the media, the deformation and evaluation of the signal in time and space take place and on the receiver we get a deformed signal. The present article is dedicated to studying the space-time evolution of rectangular, sinusoidal, exponential and bi-exponential impulses via numerical experiment in the collisional, cold plasma. The presented method is not based on the Fourier-presentation of the signal. Analytically, we have received the general image depicting the spacetime evolution of the radio impulse amplitude that gives an opportunity to analyze the concrete results in the case of primary impulse.

I. INTRODUCTION

T

HE study of the distortion of radio impulses in a dispersing plasma began a long time ago [1], [2], although the issue is still authentic due to its practical value. The objectives on the distribution of radio impulses through the ionosphere are also very important. In this direction, studies take into account the analysis of the frequency spectrum data of the scattered signal, which can be obtained through the analytical or numerical Fourier transformation [3]-[6]. In many of them, narrow band linear approximation of the signal is used, which greatly simplifies the analysis of distortion or Gaussian rectangular impulse analysis. Usually, there are many works that are devoted to the impulse propagation in non-collisional plasma, but in the communications, broadband linear impulses are mostly used during locating and sounding the surrounding environment, and therefore, the approximate assumptions made for the narrowband linear signals are unacceptable and unused. Fourier reversal transformation for wideband

linear impulses can be done only with numerical methods. The results are partially presented in works [7]-[11].

II. PROBLEM STATEMENT: RECEIVE A GENERAL IMAGE FOR THE IMPULSE ENVELOPE

Let us say we have an impulse, the propagation of which is

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described as a wave equation [12]:

$$\Delta^2 E - \text{grad div} E - \Delta^2 P = -4\pi \rho \quad (1)$$

E is tension of electric field, C is the speed of light where propagation in the media, z is the direction of the radio impulse propagation, t is the time of propagation, and P is the polarization of the unit volume in the media. Let us assume that media contains free charges, then the polarization P satisfies the equation [12]:

$$\Delta^2 P - \text{grad div} P - e_2 N = -\frac{E}{m} \quad (2)$$

where e and n are the charge of the electron and mass, N is the electronic concentration, and ν is the effective frequency of collision, which envisages the loss of energy during the collision with the ions of electrons and neutral molecules.

On the boarder of $z = 0$ half-plane, on which an impulse is propagated, a field is created, the tension of which can be represented in the following way:

$$E(z, t) = A(z, t) \exp(i\omega t), \quad t > 0 \quad (3)$$

where $z = z_0 f$; f is a carrier signal of the impulse. $A(z, t)$ is an impulse envelope, when $z = 0$. It is obvious that the impulse is propagated by C speed, so it is advisable to look for the $E(z, t)$ field in the following form.

$$A(z, t) = \exp(-i\omega t) \exp(-i\omega z/c) \quad (6)$$



$$E(z,t) = A(z,t) \exp[i\omega t - kz] \quad (4)$$

Consider in (1) and (2) that:

$$z = z_0 + ct \quad (5)$$

Insert (4) in (1) and (2) and we receive:

$$\frac{\partial^2 A}{\partial t^2} + \frac{\partial^2 A}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -\frac{e^2 N}{m} A(z,t) \exp(i\omega t) \quad (7)$$

Let us say the duration of impulse t_i - satisfies the inequality:

$$ft_i \ll 1 \quad (8)$$

and the impulse z_0 takes an interval on the axis:

$$L_i \ll ct_i \quad (9)$$

Therefore, the following condition is fulfilled:

$$\frac{\partial^2 A}{\partial z^2} \ll \frac{1}{L_i^2} A \quad (10)$$

The (10) condition allows to ignore the first member in the image (6). Since the location and speed of the electron cannot be changed immediately, at the time of transmitting impulse in the cold plasma, the following equation is fulfilled in the z_0 point:

$$P(z_0; \omega) = \frac{1}{\omega} \frac{\partial P}{\partial t} \bigg|_{t=0} \quad (11)$$

By the strength of (11), the solution of (7) can be written as the following:

$$P(z_0; t) = \frac{e^2 N}{m} \int_0^t E(z_0; \tau) d\tau$$

$$\exp\left[-\frac{z_0}{c} \left(\omega - \sqrt{\omega^2 - \frac{4e^2 N}{m}}\right)\right] \quad (12)$$

Insert the solution of (12) in (6) and take into consideration (10), then for the impulse envelope, the equation becomes:

$$\frac{\partial^2 A}{\partial z^2} + \frac{\partial^2 A}{\partial t^2} - \frac{e^2 N}{m} A(z_0; t) = 0 \quad (13)$$

For (13), let us use the Laplace transformation towards the t variable, we get [13]:

$$L\left[\frac{\partial^2 A}{\partial z^2}\right] + p^2 L[A] = \frac{e^2 N}{m} L[A] \quad (14)$$

where the following indications are introduced:

$$L\left[\frac{\partial^2 A}{\partial z^2}\right] = \frac{\partial^2 L[A]}{\partial z^2}, \quad L\left[\frac{\partial^2 A}{\partial t^2}\right] = p^2 L[A], \quad L\left[\frac{e^2 N}{m} A\right] = \frac{e^2 N}{m} L[A]$$

where, $\frac{e^2 N}{m} = \frac{1}{2} \omega_{pe}^2$, $\omega_{pe}^2 = \frac{4\pi e^2 N}{m}$ [5]; when $t = 0$ impulse still is not in the media, and therefore $A(z_0; 0) = 0$. Now, let us make Laplace's Reverse Transformation:

$$A(z_0; t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} L[A] e^{pt} dt = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{pt}}{p^2 - \frac{e^2 N}{m}} dp \quad (15)$$

$$\exp\left[-\frac{z_0}{c} \left(\omega - \sqrt{\omega^2 - \frac{4e^2 N}{m}}\right)\right] A(z_0; t) = d$$

where, $J_k(x)$ is Bessel function [13].

III. SPATIAL-TIME AND SPATIAL-FREQUENCY EVOLUTION FOR BI-EXPONENTIAL, SINUSOIDAL AND RECTANGULAR PULSE SIGNALS IN DISPERSIVE PLASMA

Let us write down the solution of (15) for the initial envelopes which have **bi-exponential** form:

$$A(z, t) = A_0 \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (16)$$

where, τ_1, τ_2 are time constants.

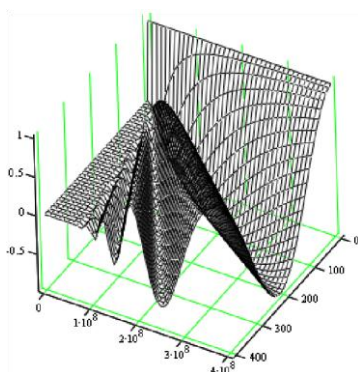


Fig. 1 The spatial-frequency evolution of the real and imaginary parts of the impulse envelope in collisional isotropic plasma when $\omega_0 = 10^7$ Hz, $\omega_1 = 400$ m, $\omega_2 = 4 \times 10^8 \text{sec}^{-1}$

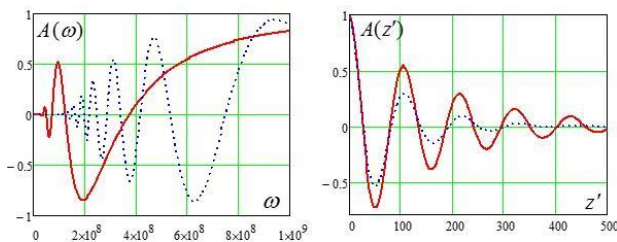


Fig. 2 (a) $A(\omega)$ attitude for different values of distance $z = 100$ m

(red line) and $z = 1000$ m (dotted line), when $\omega = 10^7$ Hz; (b) $A(z)$ attitude for different values of collision frequency $\omega = 10^7$ Hz (red line) and $\omega = 2 \times 10^7$ Hz (dotted line)

Here, A_0, τ_1, τ_2 numbers determine the curvature of the front and rear fronts of the impulse. Let us introduce a new variable $\sqrt{\tau/t}$ and insert (16) in (15). We get:

$$A(z, t) = A_0 \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (17)$$

where:

$$A(z, t) = A_0 \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (18)$$

$$A(z, t) = A_0 \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (19)$$

$$A(z, t) = A_0 \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (20)$$

$$A(z, t) = A_0 \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (21)$$

Consider the occasion when the initial impulse is of a **sinusoidal** form, then its initial envelope can be written down as following:

$$A(z, t) = A_0 \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (20)$$

$$A(z, t) = A_0 \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (21)$$

Let us introduce (19) as follows:

$$A(z, t) = A_0 \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (21)$$

where

$$g(z, t) = \exp\left[-\frac{t}{\tau_1} - \frac{t}{\tau_2}\right] \exp\left[-i\omega_0 t + i\omega_1 t + i\omega_2 t\right] \quad (21)$$

Let us put (21) in (15) and do the same operations as above, then for the envelope of a deformed sinusoid impulse we get:

$$A_z(t) = g(z, t) \exp(-\alpha z) \exp(-i\omega t) \quad (22)$$

where:

$$g(z, t) = J_1(2x\sqrt{t}) \exp(-\alpha z) \exp(-i\omega t) \quad (23)$$

Now, go to the case of **rectangular** impulse envelope:

$$A_0(t) = \begin{cases} A_0 & 0 \leq t \leq t_0 \\ 0 & t > t_0 \end{cases} \quad (24)$$

After the distortion, such impulse can be described as:

$$A_z(z, t) = g(z, t) \exp(-\alpha z) \exp(-i\omega t) \quad (25)$$

In the case of finding $g(z, t)$ function, we should pay attention to the fact that, at the same time, when $t \rightarrow 0$ and $z \rightarrow 0$, the bi-exponential impulse is transformed into a stepfunction signal with the height of A_0 , at the same time $A \rightarrow 0$, when $t \rightarrow 0$. In (18) and (19) expressions after performing the corresponding transformations, when we

get $t \rightarrow 0$, and we get the envelope of the step-function signal $A_0 \exp(-\alpha z)$:

$$g(z, t) = \exp(-\alpha z) \exp(-i\omega t) \quad (26)$$

It should be noted that each of the above solutions can be presented as a combination of Lommel's function [14]. With $t \rightarrow 0$ growing, the task on the fall of the step-function signal on the semi-infinite boundary of homogeneous media is simplified and transfers onto the task of propagating the flat wave in a homogeneous media. Let us show that the obtained result satisfies this requirement. Apply (15) that takes the following image for the step-function signal:

$$A_z(z, t) = \exp(-\alpha z) \exp(-i\omega t) \quad (27)$$

It is easy to notice that in (26) and (27) the correlations are equal. In the last equation $t \rightarrow 0$; then the integral on the right side will be reduced to a tabular form. According to its calculations, we get:

$$A_z(z, t) = \exp(-\alpha z) \exp(-i\omega t) \quad (28)$$

where, $\alpha(\omega) = \omega^2 \mu_p / (c^2)$ - is plasma absorption coefficient and $\alpha(z) = \alpha(\omega) z$ - is the optical depth on the z distance the wave passes through.

In Fig. 1, the space-frequency evolution of the impulse envelope is represented as a three-dimensional schedule. The real part indicates at the change of impulse amplitude, and the imaginary - at the reduction, which is caused by relaxation processes. In this case, $\omega = 400$ m, $\omega = 4 \cdot 10^8 \text{sec}^{-1}$. For better observation on the picture, the value $\omega = 10^7$ Hz has been chosen for frequency of the collision.

Fig. 2 shows $A(z)$ and $A(z)$ attitudes on the plane in the section. In Fig. 2 (a), it is possible to see the frequency dispersion effect, in particular, during value of $z \approx 100$ m (unbroken line), the envelope aspires towards saturation along with the frequency increase or when $z \approx 1000$ m (dotted line), additional oscillations generate the amplitude of which increases slowly along with the frequency increase. From Fig. 2 (b) it is clear that the increase of the frequency of collisions causes the rapid reduction of oscillations of impulse; with the purpose of good visualization, we have brought here the values of the frequencies of $\approx 10^7$ Hz (dotted line) and $\approx 2 \cdot 10^7$ Hz (red line) collisions.

IV. EVALUATION OF THE IMPULSE PROPAGATION RATE IN THE DISPERSIVE PLASMA

Let us find the analytical expression for the speed of the radio impulse propagation. Let us use (26) and calculate the integral applying Bessel's well-known ratio:

$$\int_0^\infty x^k J_k(x) dx = \frac{1}{2} \int_0^\infty x^{k+1} J_k(x) dx \quad (29)$$

As a result we receive:

$$g(z, t) = \exp(-\alpha z) \int_0^\infty J_k(x) \exp(-x^2) dx \quad (30)$$

The following equality will always be on the ionospheric track: $2\sqrt{\alpha z} \approx 1$.

Let us apply the asymptotic representation of the Bessel's function, for the greater value of the argument [14]:

$$J_k(x) \approx \sqrt{\frac{2}{\pi x}} \cos\left(2\sqrt{x} - \frac{k}{2} - \frac{\pi}{4}\right) \quad (31)$$

Insert (31) in (30), then for $A(z, t)$ to get the expression:

$$A(z, t) = \frac{1}{\sqrt{z}} \exp(-\alpha z) \int_0^\infty \frac{\cos\left(2\sqrt{x} - \frac{k}{2} - \frac{\pi}{4}\right)}{\sqrt{x}} dx \quad (32)$$

From (32), we can see that on z axis, the impulse is concentrated near the point the coordinate of which satisfies the condition:

$$z \approx \frac{1}{4\alpha^2} \quad (33)$$

From here, we receive that the front of the rectangular impulse propagates with the speed:

$$v = \frac{c}{2\alpha} \quad (34)$$

Equation (34) shows that the effects of collision effect on the speed of the impulse speed, when $\alpha \approx 1$, i.e. while the impulse is strongly absorbed during propagation from the plasma media.

With the minimum losses for the impulse propagation, it is necessary to fulfill the following condition $\alpha \approx v$, therefore:

$$v \approx \frac{c}{2\alpha} \quad (35)$$

Equation (35) shows that the time required to overcome the distance by the impulse on a fixed length increases as a result of the frequency decrease and the increase of electronic concentration.

Fig. 3 shows the change of impulse speed in collisional plasma according to a frequency for different number of collisions. The wave propagates if the condition $f \approx 1$ is fulfilled, when $\alpha \approx f$ the speed of the signal propagation is minimal, as the absorption processes are important.

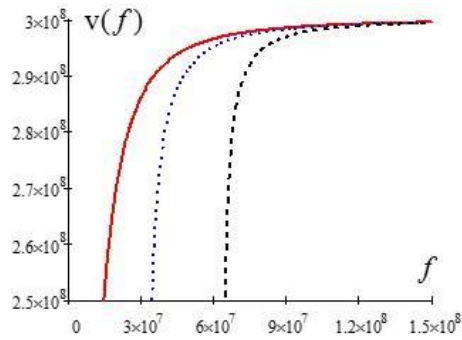


Fig. 3 Change of impulse speed according to a wave frequency during different frequencies of collisions $\square \square 10^7$ Hz (red line);

$\square \square 2 \cdot 10^8$ Hz (blue dotted line); $\square \square 4 \cdot 10^8$ Hz (black dotted line)

V. OUTCOMES OF NUMERICAL EXPERIMENT

Image (15) gives an opportunity to evaluate the change of the shape of the envelope of pulse signals of different shapes and durations in time at different altitudes from the Earth's surface. Numerical experiments are for the impulses of 10^{-4} , and 10^{-5} seconds when the frequency of the carrier signal $f \square 10$ MHz has been studied in [8]. The case of the numerical experiment when the frequency of the carrier signal is close to the plasma frequency, i.e. when: $f_0 \square 2.84$ MHz ($f \square 3$ MHz) is represented [14], [15], it is obvious that the condition $f \square f_0$ must be fulfilled, otherwise the impulse cannot be propagated in the plasma layer.

Suppose the impulse signal emits from the source of the Earth's surface. The objective of the current research is to determine the change of the impulse shape (distortion) in the

lower layers of the ionosphere at the altitude of $z \square 100$ $\square 1000$ km. Numerical calculations were carried out for the following parameters of the plasma media: Number of collisions $\square \square 10^3, 10^4 \text{ sec}^{-1}$, concentration - $N \square 10^5$.

We have studied the relationship of the radiation power A^2 on the time at different distances from the source, for different forms of impulse. It has been established that an increase in the number of collisions causes the distortion of the shape of the impulse, which is naturally explained by the emergence of dissipation processes.

Fig. 4 shows the change of the sinusoidal impulse envelope in the dispersive plasma layer at different altitudes. The impulse of $t_i \square 10^{-3}$ sec of the duration at the altitude of $z \square 200$ km still maintains the sinusoidal shape, and by the increase of the height ($z \square 700$ km), the cyclical oscillations will be created in the impulse envelope. The back front of the impulse is basically distorted. And by shortening the impulse, on the contrary, the oscillations disappear and the displacement of the maximum can be noticed. Together with the rise of collisions, these oscillations disappear, but the impulse is sharply narrowed and the amplitude falls.

It should be noted that among the discussed impulses, the bi-exponential impulse can be distinguished by its steady shape, which is different from others with less oscillations than rectangular or sinusoidal impulses. In this case, the dissipative processes caused by the collisional effects are manifested primarily in a decrease in the intensity of the pulse signal.

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