

Investigating Gravity Gradient Instrument Dynamics through Mathematical Modeling

Emily J. Patel, Liam T. Reed

Department of Aerospace Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA; Department of Physics, University of California, Berkeley, CA, USA

Abstract—Gravity field is of great significance in geoscience, national economy and national security, and gravitational gradient measurement has been extensively studied due to its higher accuracy than gravity measurement. Gravity gradient sensor, being one of core devices of the gravity gradient instrument, plays a key role in measuring accuracy. Therefore, this paper starts from analyzing the working principle of the gravity gradient sensor by Newton’s law, and then considers the relative motion between inertial and non-inertial systems to build a relatively adequate mathematical model, laying a foundation for the measurement error calibration, measurement accuracy improvement.

I. PREFACE

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ACH gravity field is a kind of cohesive field formed by the superposition of the Earth’s gravitational field and centrifugal force field. Gravity field information has important applications in geodesy, geophysical science, seismic analysis, earth exploration, inertial navigation, etc. [1]-[4]. There are many ways to obtain gravitational fields. Gravity measurements usually use gravimeters, but the accuracy and resolution of gravimetric measurements are far from meeting the requirements of exploration geophysics. The gravitational gradient meter can measure the second order derivative of the gravitational force; that is, has a higher resolution than the gravimeter. At the same time, the gravitational gradient is a measure of the rate of change of the spatial gravity field, which is less affected by the acceleration of the motion carrier [5]. Therefore, at the beginning of this century, the gravity gradient instrument was introduced into the field of resource exploration, and the gradient measurement method has also been extensively studied. The performance of traditional measurement method is poor, while the superconductivity and atomic technology exploration are beyond practical applications [6]-[8]. In this paper, the mathematical model of the method is used to analyze the input and output relations of the accelerometer according to the measurement principle, and the gradient component measurement equation is obtained, laying the foundation for the work in further mechanical

precision design, physical design, measurement and extraction of the signal.

II. CONCEPTION OF GRAVITY GRADIENT REVIEW STAGE

The physical meaning of the gravitational gradient is used to describe the change in the gravity component due to the position, and the gravitational gradient is the second derivative of the gravitational force,

$$g_x = \frac{\partial^2 W}{\partial x^2}, \quad g_y = \frac{\partial^2 W}{\partial y^2}, \quad g_z = \frac{\partial^2 W}{\partial z^2}$$

$$g_{xy} = \frac{\partial^2 W}{\partial x \partial y}, \quad g_{yx} = \frac{\partial^2 W}{\partial y \partial x}, \quad g_{yz} = \frac{\partial^2 W}{\partial y \partial z}, \quad g_{zy} = \frac{\partial^2 W}{\partial z \partial y}$$

$$g_{zx} = \frac{\partial^2 W}{\partial z \partial x}, \quad g_{xz} = \frac{\partial^2 W}{\partial x \partial z}$$

where \square is the gravity gradient tensor:

$$\square = \begin{bmatrix} \frac{\partial^2 W}{\partial x^2} & \frac{\partial^2 W}{\partial x \partial y} & \frac{\partial^2 W}{\partial x \partial z} \\ \frac{\partial^2 W}{\partial x \partial y} & \frac{\partial^2 W}{\partial y^2} & \frac{\partial^2 W}{\partial y \partial z} \\ \frac{\partial^2 W}{\partial x \partial z} & \frac{\partial^2 W}{\partial y \partial z} & \frac{\partial^2 W}{\partial z^2} \end{bmatrix}$$

There are nine components of the gravity gradient tensor:

$\begin{matrix} \cdot_{xx} \\ \cdot_{yy} \\ \cdot_{zz} \\ \cdot_{xy} \\ \cdot_{yx} \\ \cdot_{yz} \\ \cdot_{zy} \\ \cdot_{xz} \\ \cdot_{zx} \end{matrix}$, among them \cdot_{xx} represents for the g_x space rate of change in x direction, \square_{xy} represents for the g_x space rate of change in y direction.

$$\square_{xy} = \frac{\partial^2 W}{\partial x \partial y}, \quad \square_{yx} = \frac{\partial^2 W}{\partial y \partial x}, \quad \square_{yz} = \frac{\partial^2 W}{\partial y \partial z}, \quad \square_{zy} = \frac{\partial^2 W}{\partial z \partial y}$$

Outside the earth, as the gravitational potential is continuous and limited everywhere, gravity potential meets the Laplace equation:

$$\begin{aligned}
 & \cos^2 t \left[a_{ox} \cos 2t + a_{oy} \sin 2t \right] \\
 & \frac{1}{2} \left[a_{ox} \cos 2t + a_{oy} \sin 2t \right] \\
 & \frac{1}{2} \left[a_{ox} \cos 2t + a_{oy} \sin 2t \right]
 \end{aligned}$$

where t belongs to accelerometer one while that of number two, three and four are t_2 , t_3 , and t_4 , respectively. The following equations tell the relationship between the direction vectors of the sensitive axis e_1, e_2, e_3, e_4 :

$$\begin{aligned}
 e_1 &= \cos t \cdot o_1 + \sin t \cdot o_2 \\
 e_3 &= \cos t \cdot o_1 - \sin t \cdot o_2 \\
 e_2 &= \cos t \cdot o_1 + \sin t \cdot o_2 \\
 e_4 &= \sin t \cdot o_1 - \cos t \cdot o_2
 \end{aligned} \tag{9}$$

Make a combination and the result leads to the components of the gravity gradient:

$$\begin{aligned}
 & a_{11} \cdot e_1 + a_{22} \cdot e_2 + a_{33} \cdot e_3 + a_{44} \cdot e_4
 \end{aligned}$$

Get the following equation after the force analysis of the block:

$$m \ddot{x} + c \dot{x} + kx = F(t) \tag{11}$$

$$\dots \tag{10}$$

B. Mathematical Model of the Working Principle

Although the rotational accelerometer gravity gradient instrument is based on the same principle, differences in the mathematical working model are derived in literature [8], [11]-[14]. That the measurement model is perfect or not, is a serious influence on the accuracy of the measurement, as the gradient signal is extraordinarily weak. Strictly speaking, the platform system of the gravity gradient sensor, the carrier system and even the earth, are not the inertial coordinate system. In this paper, force analysis between the inertial and non-inertial systems is taken into consideration, as well as the tested acceleration and the actual gradient values of the disc center to establish the correlation model. Therefore, a more complete measurement model comes into conclusion, laying a good foundation for error model analysis, mechanical precision design, and the derivation of carrier platform stability requirements.

Based on the kinetic force analysis, the block of accelerometer moves from N to B at the equilibrium position when the spring force and the damper resistance are zero, and the internal structure is shown in Fig. 1. In this moment, the spring force is balanced to the inertial force, and also by the universal gravitation.

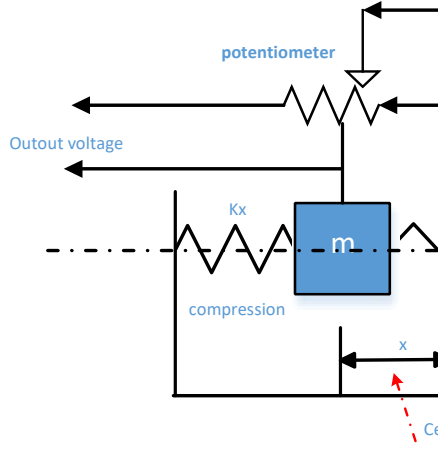


Fig. 2 Basic structure of t

is twice the speed of the turntable, and where the gravitational gradient signal can be further demodulated.

V. CONCLUSION

In this paper, the concept of gravity gradient information is introduced in detail, and the mathematical model of the input and output of gradient sensor is established by analyzing the principle of gravity gauging. Each gravitational gradient sensor can demodulate two equations on the gravitational gradient signal. Therefore, by installing three gravitational gradient sensors that orthogonal to each other, can all the components of the gravitational gradient be derived, for six different equations can be demodulated from the signal. This model comprehensively covers the factors that affect the output of the gravity gradient sensor, which lays a foundation for deducing the measurement error of the gravity gradient sensor and compensate for the error. It is also the basic for proposing the stability of the sensor structure and the stability requirement of the carrier platform.

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