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Analyzing Geometric Properties and Neighborhoods in Multivalent Function Theory

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***Abstract*—**By using the two existing operators, we have defined an operator, which is an extension for them. In this paper, first the operator is introduced. Then, using this operator, the subclasses of multivalent functions are defined. These subclasses of multivalent functions are utilized in order to obtain coefficient inequalities, extreme points, and integral means inequalities for functions belonging to these classes.

I. INTRODUCTION

W

E have defined the operator , , , which is an extension of operators of Al-Oboudi Operator [1] and Sãlãgean Operator [9]. Also we define two subclasses , , , , , and , , , , , using the above mentioned operator. These operator's subclasses mentioned are extension of , , , and , , , , which are introduced by Eker and Seker [5].

Let be the class of functions of the form

$$\sum_{j=p+1}^{\infty} a_j z^j, \quad (p \in \mathbb{N} = \{1,2,3,\ldots\}) \quad (1)$$

which are analytic and -valent in the open unit disc ∈ : | | 1 .

Definition 1. Let , ∈ , 0, 0, , ∈ and

$$\sum \quad .$$

Then, we define the operator as: , , : → by

$$, \, , \quad \sum \quad 1 \quad \Big] \quad {}^{j}, \quad (z \in \mathcal{U}). \quad (2)$$

Definition 2. The function ∈ is to be in the class , , , , , if

$$11 \quad -\Big[\quad , \, , \quad , \, , \quad \Big[\quad , \, , \quad , \, , \quad 1 \quad (3)$$

for some 0 1, 0, 0, , ∈ \ 0 and ∈ . 0, ∈ , ∈

Let be the subclass of consisting of

$$f z \quad z \quad \sum \quad a z, \quad a \quad 0, \; p \in \quad , \quad (4)$$

which are p - valent in . Let Ω be the class of function ω analytic in such that ω 0 0, |ω z | 1.

For any two functions f and g in , f is said to be subordinate to g denoted f ≺ g if there exists an analytic function ω ∈ Ω such that f z g ω z [2]. Further, if f z z ∑ a z , then we have :

$$D \quad \lambda, q, n f z \quad z \quad \sum \quad 1 \quad \Big] a z, \quad z \in \quad . \; (5)$$

For any function f ∈ and δ neighborhood 0, the δ - of f is defined s :

$$f \quad g z \quad z \quad \sum \quad b z \in \; : \sum \quad j |a \quad b| \quad \delta . (6)$$

In particular, for the function e z z , we see that :

$$e \quad g z \quad z \quad \sum \quad b z \in \; : \sum \quad j |b| \quad \delta . (7)$$

The concept of neighborhoods was first introduced by Goodman [6] and then generalized by Ruscheweyh [8].

Definition 3. A function f ∈ is said to be in the class β, n, m, p, q, A, B, λ if

$$, , \quad \prec \quad , \quad z \in \; , \quad (8) \quad , ,$$

where, γ 1 β A βB, 0 β 1, q, n ∈ , m ∈ , λ 1 and 1 B A 1.

II. COEFFICIENT INEQUALITIES FOR THE CLASS , , , , ,

Theorem 1. If ∈ satisfies, then

$$\sum \quad , \, , \, , \, , \, , \, , \quad 2| \quad | \, 1 \quad (9)$$

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where,

, , , , , , ,
1

$$\left[1+\frac{(j-p)\lambda}{p+q}\right] \quad 1$$

$$\frac{(j-p)\lambda}{p+q}\Big]^{n}+\left[1+\frac{(j-p)\lambda}{p+q}\right]$$

—— 2 1 1 $\frac{(j-p)\lambda}{p+q}\Big]^{n}\Big|\,\big|a_j\big|\geq 0.$

2 1 —— 1

III. RELATION FOR , , , , ,

By Theorem 1 we introduce the class , , , , , as the subclass of , , , , , consisting of satisfying

$$\sum \quad , \; , , , , , , \qquad 2| \; |1 \qquad (10)$$

where,

, , , , , , ,

$$\frac{(j-p)\lambda}{p+q}\Big] - \left[1+\frac{(j-p)\lambda}{p+q}\right]$$

$$b(2-\alpha)-1]\left[1+\frac{(j-p)\lambda}{p+q}\right]^{n}$$

$$\frac{(j-p)\lambda}{p+q}\Big]^{m} - \left[1+\frac{(j-p)\lambda}{p+q}\right]$$

1 1

21

2 1

for some 0 1 , 0, ∈ , ∈ and ∈ \ 0 .

Theorem 2. If ∈ , then , , , , , ⊂ , , , , , for some and , such that

0 .

Proof. For 0 , we have

$$\sum \quad , \; , , , , , ,$$
$$\sum \quad , \; , , , , , , \quad .$$

Therefore, if ∈ , , , , , , then ∈ , , , , , .

IV. EXTREME POINTS

The determination of extreme points of a family of univalent functions enables us to solve many external problems for .

Theorem 3. Let and

| |

—————— , 1, 2, ... ; 1 .

, , , ,, , , ,

Then, ∈ , , , , , if and only if it can be expressed in the form

∑ ,

where, 0 and 1∑ .

Proof. Suppose that

| |

∑ ∑ —————— .

, , , ,, , , ,

Then,

| |

∑ , , , , , , , ——————

, , , ,, , , ,

∑ 2 | | 1
2| | 1 ∑

2| | 1 1
2| | 1 .

Thus, ∈ , , , , . Conversely, let ∈ , , , , , . Since

| | 1, 2, ...

——————

we put , , , ,, , , , , 1

, , , ,, , ,,

——————

| |

and

1∑ .

Then,

∑ .

Corollary 1. The extreme points of m, n, α, β, b, q are the functions f z z and

| | p 1, p 2, ... ; ε 1 .

f z z ——————z , j

, , , ,, , ,,

V. DISTORTION AND COVERING THEOREMS

Theorem 4. If f □T(□,n,m,p,q,A,B,□) , then

$$r^p \cdot \frac{\cdot\ \cdot\ B}{(1\cdot \frac{1}{p\cdot q}\cdot)^n \cdot (1\cdot B)(1\cdot \frac{1}{p\cdot q}\cdot)^m \cdot (1\cdot\cdot)\cdot} r^{1\cdot p} \cdot |f(z)| \cdot$$

$$r^p \cdot \frac{\cdot\ \cdot\ B}{(1\cdot \frac{1}{p\cdot q}\cdot)^n \cdot (1\cdot B)(1\cdot \frac{1}{p\cdot q}\cdot)^m \cdot (1\cdot\cdot)\cdot} r^{1\cdot p} .3cm(0<|z|=r<1),$$

with equality for

$$f(z) = z^p □ \frac{□□ B}{\frac{1}{□}^n □ □ (1□ \frac{1}{p□q}□)^m □(1□ B)(1□ □) □ (1□□)□ p □ q □} r^{1□p} .3cm(z=□r) \qquad (11)$$

Proof. In Theorem 1, we have

$$□3.5cm(1□ \frac{1}{\ } □)^n □ □(1□ B)(1□ \frac{1}{\ } □)^m □(1□□)^□ □ \boxed{\ }^{□} a_k p □ q □ \ p □ q \ □_{k=1□p}$$

$$□ □(k□ p)□□^{n} □^□ □ \ □ (k□ p)□□^{m} □^□ □ \ □ \boxed{\ }_{=1□} □1□ \frac{\ }{p□q} □□ □□□(1□B)□□1□ \frac{\ }{p□q} □□ □(1□□)□□□a_k□□□B.$$

$k \quad p□$

Hence

$$\frac{1}{\ }^{n} \quad \frac{1}{\ }^{m}$$

$$\cdot 3.7cm\,|f(z)| \cdot r_p \cdot \bullet\ a_k r_k \cdot r_p \cdot r_{1\cdot p} \bullet\ a_k$$

$k=1\cdot p \qquad k=1^{p}$

$$1cm \cdot r_p \cdot \frac{\cdot\cdot B}{(1\cdot \frac{1}{p\cdot q}\cdot)^n \cdot (1\cdot B)(1\cdot \frac{1}{p\cdot q}\cdot)^m \cdot (1\cdot\cdot)\cdot} r_{1\cdot p}$$

and

$$\cdot 3.7cm\,|f(z)| \cdot r_p \cdot \bullet\ a_k r_k \cdot r_p \cdot r_{1\cdot p} \bullet\ a_k$$

$k=1\cdot p \qquad k=1\cdot p$

$$\frac{\cdot\cdot B}{(1□ \quad □) □□(1□B)(1□ \quad □) □(1□□)^□ □ p□q □ \quad p□q □} 1cm \cdot r_p \cdot r_{1\cdot p}.\ k^{\bullet} p$$

This completes the proof.

Theorem 5. Any function f □T(□,n,m,p,q,A,B,□) maps the disk $|z|<1$ on to a domain that contains the disk

$$\frac{□□ B}{\frac{j}{\ }^n □ \quad □(i□ \ □)^m □(1□ B)(1□ \quad □) □ (1□□)□ p □ q □ \quad p □ q □} \quad |w|<1□.$$

Proof. The proof follows upon letting $r□1$ in Theorem 4.

Theorem 6. If f □T(□,n,m,p,q,A,B,□) , then

$$1 □ \frac{(□□ B)}{□(1□ B)(1□ \quad □) □ \quad □(1□ \quad □) \quad □}$$

$f(z)=z_p$

Proof . V

In Theorer

ka

$r\cdot|f\cdot(z)|\cdot$

$\frac{1}{}$ $^{n\cdot 1}$ $\frac{1}{}$ ^{m}

(1□□)□ p □ q □ p □ q

□

□□ B ^{1}

1 □r.3cm(0 <| z |= r <1),

$\frac{1}{}$ $^{n\cdot 1}$□ □1(1□ ^{m} □)

□(1□ B)(1□ □) □

(1□□)□ p □ q □ p □ q

□

with the equality for

=1□

(1□ □) □(1□B)(1□ □)

□(1□□)□ p□q □ p□q □

Thus

$$\overline{(1\cdot\frac{1}{p\cdot q}\cdot)^{n\cdot 1}\cdot(1\cdot\quad)(1\cdot\frac{1}{p\cdot q}\cdot)^{m}\cdot(1\cdot\cdot)\cdot}\quad .$$

On the other hand,

| f □(z) |□1□ □□ B r.

□ □

| f □(z) |□1□ □ ka_k| z |$^{k□p}$□1□r □ ka_k

k=1□p k=1□p

□□ B

□1□r.

(1□ 1 □)$^{n□1}$□□(1□ B)(1□ 1

□)^{m} □ (1□□)□□ p □ p

□ q □

This completes the proof.

VI. NEW SUBCLASSES OF RADII OF STARLIKENESS AND CONVEXITY

We calculate the radius of starlikeness of order □ and the radius of convexity of order □ for functions T(□,n,m, p,q, A, B,□) .

Theorem 7. Let f □T(□,n,m, p,q, A, B,□) , then we have

f as starlike of order □,(0 □□< p) in | z |< r_1(□,n,m, p,q, A,B,□,□) where

r_1(□, ,n m p q A B, □□ (k p□)^n□□ □□ □ (k p□)^m□□ □□ □k p□ ^{1}

, , , , ,□□) = inf $_k$ ~~□□□~~ p q□ □□ □.1cm□□□(1□ B) 1□□

~~□ p q□ □□ □.18cm□□.18cm(1□□) (□□□ p □□) □.1cm□~~□ □

□□1□

□ (k □□□)(□ B) □

□ □

□□ □□

Proof. We show that z ff□((zz)) □ p □ p □□(0 □ □< p) for □□(k $\left|z\frac{f\cdot(z)}{f(z)}\cdot p\right|\cdot$ $\frac{k=1\cdot p \quad k}{\cdot\ a\ |z|^{k\cdot p}}$ □ p)a | z |$_{k□p}$

| z |< r_1(□,n,m, p,q, A, B,□,□) .

We have 1□ ^{k}

k=1□p

Thus

z ff□((zz)) □ p □ p □□ | z □□□□1□

□.18cm□□.18cm(1□□)□□□□□(p□□) □.1cm□□□□k□1 p

□ (k □□)a_k | z |k□p

□ (p□□) □1. (13)

k=1□ p

Then by Theorem 1, will be true if

| z |< r₂(□,n,m, p,q, A,B,□,□) where n m

(k□p)□□ □ (k□p)□□

(1□B) 1□

p□q

or if

(kp□□pq)□□□□n □.1cm□□□□(1□B)□□□1□ (kp□□pq)□□□□m

□□

|□□

□ (k□□)(□□B) □

□ □ if □□ □□

(k □1□ p). (14)

Theorem 8. Let f □T(□,n,m, p,q, A, B,□) , we proof that

f is convex of order □,(0 □□< p) in

1

(k p□)□□

□

□□

(k □1□ p).

T(□,n,m, p,q, A, B,□) if and only if

p q

(16)

where,

and

□1□ B < A □1.

f

Dp(□,q,m□n) f (z) 1□□z z□U.

□ (17)

(□□B)

Therefore, there exists an analytic function □ such that or if

1

□□ 1□ (k □ p)□□n □ .1cm (1□ B) 1□ (k □ p)□□m □ .18cm□□.18cm(1□□) (p□□)□□k □ p □(z) =

Dp(□,q,n) f (z)□ Dp(□,q,m□n) f (z)

| z |□□□ p□q p□q □ .1cm□□ BDp(□,q,m□n) f (z)□□Dp(□,q,n) f (z) (18)

□□ k(k □□)(□□B) □□

□□ □□ Hence,

□(z) = Dp(□,q,n)f (z)□Dp(□,q,m□n)f (z)

BDp(□,q,m□n)f (z)□□Dp(□,q,n)f (z)

n m

□□ (j□ p)□□ □□□ (j□ p)□□ □□

j

| | □ □1□ jp □

p□q □□

□□□1□

= = □1 □□ np□q □□ □1□□□amjz

<1.

□□ (j□ p)□□ □□ □ (j□ p)□□□□

j

(□□B)zp □ j □ =p□1□□1□ p□q □□

□□□B□□1□ p□q □□ □□□□□ajz

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Thus,

$$\Re\left\{\frac{\sum_{j=p+1}^{\infty}\left[1+\frac{(j-p)\lambda}{p+q}\right]^{n}\left\{\left[1+\frac{(j-p)\lambda}{p+q}\right]^{m}-1\right\}a_jz^j}{(\gamma-B)z^p+\sum_{j=p+1}^{\infty}\left[1+\frac{(j-p)\lambda}{p+q}\right]^{n}\left\{B\left[1+\frac{(j-p)\lambda}{p+q}\right]^{m}-\gamma\right\}a_jz^j}\right\}<1.$$

Taking $|z|=r$, for sufficiently small r with $0<r<1$, the denominator of (19) is positive and so it is positive for all r with $0<r<1$, since $\omega(z)$ is analytic for $|z|<1$. Then, the inequality (19) yields

$$\sum_{j=p+1}^{\infty}\left[1+\frac{(j-p)\lambda}{p+q}\right]^{n}\left\{\left[1+\frac{(j-p)\lambda}{p+q}\right]^{m}-1\right\}a_jr^j$$

$$<(\gamma-B)r^p+B\sum_{j=p+1}^{\infty}\left[1+\frac{(j-p)\lambda}{p+q}\right]^{m+n}a_jr^j-\gamma\sum_{j=p+1}^{\infty}\left[1+\frac{(j-p)\lambda}{p+q}\right]^{n}\ {}_j$$

Equivalently,

$$\sum_{j=p+1}^{\infty}\left[1+\frac{(j-p)\lambda}{p+q}\right]^{n}\left\{(1-B)\left[1+\frac{(j-p)\lambda}{p+q}\right]^{m}-(1-\gamma)\right\}a_jr^j\le(\gamma-B)r$$

and (16) follows upon letting $r\to1$. Conversely, for $|z|=r, 0<r<1$, we have $r^j<r^p$. That is,

$$\sum_{j=p+1}^{\infty}\left[1+\frac{(j-p)\lambda}{p+q}\right]^{n}\left\{(1-B)\left[1+\frac{(1-p)\lambda}{p+q}\right]^{m}-(1-\gamma)\right\}a_jr^j$$

□□ n p□q □□□

□

□

□□

(19)

ar^j.

p

□j□=□p□1□□□1□(jp□pq)□□□□□□(1□ □ (j□p)□□m □□ p B)rp.

B)[□1□

□ □□□ □ p□q □□

□(1□□)□□□ajr □(□□

From (16) we have

n m

□ □ (j □ p)□□ □□□(j □ p)□□□□aj

□ j= p□1□□1□ p□q □□ □□□□□1□

p□q □□ □1□□□ jz

□□ (j□p)□□n□□□ (j□p)□□m □□ j

□ □ □1□ p□q □□ □□□□□1□

p□q □□ □1□□□ajr j=p□1□

<(□□ ajr

B)rp □

j□=□p□1□□□□□B□□□1□(jp□□pq)□□□

□m □□□□□□□□□□1□(jp□□pq)□□□□n

j

B)z^p □ □ □

□□□B□□1□(jp□□pq)□□□□^m

□□□□□□□□□□□1□(jp□□pq)□□□□^n j

<(□□ $a_j z$.

j=p□1□□ □

This proves that

$$\frac{D_p(\square,q,m\square\ n)f(z)}{D_p(\square,q,n)f(z)} \square \frac{1\square\square z}{1\square\ Bz}, \quad z\square U$$

and hence f □T(□,n,m, p,q, A, B,□).

Theorem 10. If

$$\square = \frac{(\square\square B)}{\left(1\square\ p\ q\square\right)^{\frac{1}{n\cdot 1}}\square \square\square(1\square B)\left(1\square\ p\square q\square\right)^{\frac{1}{m}}\square(1\square\square)\square\square} \qquad (20)$$

□

then T(□,n,m, p,q, A,B,□) □ $N_\square(e)$.

Proof. It follows from (16) that if f □T(□,n,m, p,q, A, B,□) , then

$$(1\square\underline{\quad}p1q\square)_{n\square 1}\square\square(1\square B)(1\square\underline{\quad}p1q\square)_m \square(1\square\square)\square\square\square \square_{j=p\square 1}\square ja_j \square(\square\square B), \quad (21)$$

□ □ □

which implies,

$$\square_j \square_{=p\square 1} ja_j \le \frac{(\square\square B)}{\overline{\square(1\square\ p1\square q\square)_{n\square 1}}\square\square\square(1\square B)(1\square\ p1\square q\square)_m\square(1\square\square)\square\square\square} = \square. \quad (22)$$

Solving (22) we get the result.

VIII. CONCLUSION

The following are the special cases of the class

, , , , , :

1. , , , , 1,0 ≡ , „ , the class introduced by Eker and Seker [5].
2. , , , , 1,0 ≡ ‚ , , the class studied by Eker and Owa [3].
3. 1,0, , , 1,0 ≡ , and 2,1, , , 1,0 ≡ , , the classes studied by Shams et al.[10].
4. 1,0, , 0,1,0 ≡ * and 2,1, , 0,1,0 ≡ , the classes suggested by Robertson [7]. 5. , , , 0,1,0 ≡ „ the class studied by Eker and Owa [4].

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