

Fuzzy Transportation Problem Solving via Best Candidates Method and Multiple Ranking Approaches

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Abstract—Transportation Problem (TP) is based on supply and demand of commodities transported from one source to the different destinations. Usual methods for finding solution of TPs are NorthWest Corner Rule, Least Cost Method Vogel's Approximation Method etc. The transportation costs tend to vary at each time. We can use fuzzy numbers which would give solution according to this situation. In this study the Best Candidate Method (BCM) is applied. For ranking Centroid Ranking Technique (CRT) and Robust Ranking Technique have been adopted to transform the fuzzy TP and the above methods are applied to EDWARDS Vacuum Company, Crawley, in West Sussex in the United Kingdom. A Comparative study is also given. We see that the transportation cost can be minimized by the application of CRT under BCM.

I. INTRODUCTION

T

P helps in solving problems in distribution and transportation of resources from one place to another. It deals with the transportation of a single product manufactured at different origins to a different number of destinations.

The optimization processes in Mathematics, Computer Science and Economics are solved effectively by choosing the best element from set of available alternative elements. In this paper, we have used a solution technique called the BCM which is used to solve optimization problem.

Our aim is to minimize the transportation cost. Different methods used for solving TPs are trying to reach the optimal solution, whereby, most of these methods are expensive and time consuming. In this paper, we propose a Ranking technique for solving fuzzy TP, where the fuzzy demand and supply are in the form of fuzzy numbers. Here we use (BCM) in which we elect the best candidates that gives the lowest combinations to get the optimal solution.

Comparatively, applying the BCM in the proposed method obtains the optimal solution to a TP and performs faster than the existing methods with a minimal computation time and less complexity [1]-[4], [18].

A. Robust's Ranking Techniques

Robust's Ranking Techniques which satisfies Compensation, Linearity and Additive properties and provides results which are consistent with human intuition. Give a

convex Fuzzy number \tilde{a} , the Robust's ranking index is defined by $R(\tilde{a}) = 0.5 \int_0^1 \alpha d\alpha$ where (α, β) is the cut of a fuzzy number \tilde{a} and $(\alpha, \beta) = ((b-a)\alpha + a), (c-(c-b)\alpha)$ [3], [6], [14], [15].

B. Steps Involved in BCM

- Step 1:** Consider the BCM matrix. The matrix should be balanced without using the added row or column candidates in the method.
- Step 2:** To minimize the transportation cost or to maximize the profit, we have to choose the best candidates. In each row, we choose the best two candidates. A candidate should not be repeated more than two times. In this case then the candidate should be chosen again. In the same way, the columns should also be checked.
- Step 3:** For each row and column, we choose one candidate which has the minimum candidate. Start with the row that has the least candidate and delete the row and column. In case if some rows and columns do not have selected candidates, then select the best candidate from the remaining candidates. Repeat the above process till the last candidate [1], [2].

C. New Algorithm

In this study, we propose a new algorithm for TPs by using BCM. The steps involved in this algorithm are as follows:

- Step 1:** We must balance the transportation table.
- Step 2:** For each row and column find the lowest cost of the weights using the BCM.
- Step 3:** In the selected row or column where the cost candidate is low, allocate the maximum amount of supply and demand. After this, we assume the row or column to be zero. Now, we choose among the rows or columns which are not as assigned as zero, the one which has the least cost.
- Step 4:** Elect the next least cost from the chosen combination and repeat Step 3 until all columns and rows is exhausted.

To define the ranking of generalized trapezoidal fuzzy numbers, the centroid of the centroid G_1 , G_2 and G_3 is taken as the solution point. The centroid of the trapezium is obtained by using the centroid of the triangle ABP, the triangle CQD and the rectangle BPQC [11].

Consider a generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$. The centroid of these plane figures are $G_1 = (\dots)$,

$G_2 = (\dots)$ and $G_3 = (\dots)$ respectively

Equation of the line G_1, G_3 is $y = \dots$ and G_2 does not lie on the line G_1, G_3 . Thus G_1, G_2 and G_3 are non collinear and they form a triangle. We define the centroid $G_A(x_0, y_0)$ of the triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$ as $G_A(x_0, y_0) = (\dots)$. As a special case, for triangular fuzzy numbers $A = (a, b, d; w)$ i.e., $c = b$ the centroid of centroid is given by $G_A(x_0, y_0) = (\dots)$ [6], [7], [16], [11].

F. Numerical Example

If the fuzzy transportation cost for unit quantity of the product from i^{th} source and j^{th} destination is C_{ij} , where

	1, 4, 9		
			16, 25,
	36		
		9, 36, 49	
$C_{ij} =$	16, 25, 64		
	36, 64, 81		
			49, 64,
4, 49, 64			
	4, 25, 81		
		25, 36, 64	
			49, 64,
	81		

The fuzzy TP can be formulated as follows:

TABLE VI
SUPPLY AND DEMAND IN TRIANGULAR FUZZY NUMBERS

	Destination1	Destination2	Destination3	Supply
Source1	(1, 4, 9)	(16, 25, 36)	(9, 36, 49)	(4, 25, 36)
Source2	(16, 25, 64)	(36, 64, 81)	(4, 49, 64)	(16, 36, 49)
Source3	(4, 25, 81)	(25, 36, 64)	(49, 64, 81)	(25, 49, 81)
Demand	(16, 25, 36)	(4, 49, 81)	(25, 36, 49)	

Solution: Using the above Ranking method the given problem can be reduced as follows:

TABLE VII
SUPPLY AND DEMAND IN CRISP VALUE AFTER CRT

	Destination1	Destination2	Destination3	Supply
Source1	1.69	10.5	13.78	9.46
Source2	11.71	25.97	17.93	14.39
Source3	11.41	15.81	27.09	20.48
Demand	10.5	18.66	15.17	

Step 1: The given transportation table is a balanced table.
Step 2: We choose the candidate which has the least cost from each row and column, using the BCM.

TABLE VIII
SELECTION OF LEAST COST

	Destination1	Destination2	Destination3	Supply
Source1	1.69	10.5	13.7	9.46
Source2	11.7	25.97	17.93	14.39
Source3	11.4	15.81	27.09	20.48
Demand	10.5	18.66	15.17	

The least of all the values is the best candidate for each row or column.

TABLE IX
SELECTION OF BEST CANDIDATE

	Destination1	Destination2	Destination3	Supply
Source1	1.6	10.5	13.78	9.46
Source2	11.71	25.97	17.9	14.39
Source3	11.41	15.8	27.09	20.48
Demand	10.5	18.66	15.17	

Step 3:

TABLE X
ALLOCATION OF SUPPLY AND DEMAND

	Destination1	Destination2	Destination3	Supply
Source1	9.46 1.69	10.5	13.78	9.46 0
Source2	11.71	25.97	14.39 17.9	14.39 0
Source3	1.04 11.41	18.66 15.81	0.78 27.09	20.48 19.44 0.78 0
Demand	10.5 1.04 0	18.66 18.66 0	15.17 15.17 0	

The least transportation cost using BCM is $(1.69)(9.46) + (17.93)(14.39) + (11.41)(1.04) + (15.81)(18.66) + (2$

$$7.09)(0.78) = 596.55 [5], [8]$$

II. APPLICATION OF THE BCM METHOD TO THE REAL DATA

Edwards is a world leader in the manufacturing and supply of vacuum and abatement solutions in Crawley, in West Sussex in the United Kingdom. For nearly 100 years they have supported their customers by providing the clean environments required for their

processes and by continually innovating methods, the company provides equipment and services across numerous industries:

PROBLEM: Edwards – vacuum engineering company manufactures vacuum pumps for the production of scientific instruments. The quarterly demand for its products is 100, 200, 180 and 150 pumps respectively. The company can produce 80, 150, 230 and 170 pumps in four months. Pumps are transported from four distribution centers to four dealers. The mileage chart between the manufactures and the distribution centers in kilometers are given below.

TABLE XI
THE MILEAGE CHART

	Bangalore	Pune	New Delhi	Kolkata
Korea	5028	5015	5040	5035
Japan	5820	5800	5900	5850
UK	7520	7500	7600	7560
Lupton	7320	7300	7400	7380

The transportation costs per pump on different routes, rounded to the closest dollar are given below

TABLE XII
TRANSPORTATION COST

	Bangalore	Pune	New Delhi	Kolkata
Korea	75	70	85	80
Japan	86	82	96	90
UK	102	90	136	120
Lupton	100	98	115	112

The LP model of the problem is given as

$$\begin{aligned}
 &\text{Minimize } z = 75 \quad +70 \quad +85 \quad 80 \quad 86 \\
 &\quad 82 \quad 96 \quad 90 \\
 &\quad 102 \quad 90 \\
 136 &\quad 120 \quad 100 \quad 98 \quad 115 \quad 112 \\
 \\
 &\text{Subject to} \quad \quad \quad 80 \\
 &\quad \quad \quad \quad \quad 150 \\
 &\quad \quad \quad \quad \quad 230 \\
 &\quad \quad \quad 170 \quad 100 \\
 &\quad \quad \quad 200 \\
 &\quad \quad \quad 180 \\
 &\quad \quad \quad 150
 \end{aligned}$$

for $0 \leq x_{ij} \leq \min\{a_i, b_j\}$, $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$

These constraints are all equations because the total supply from the four sources ($80+150+230+170= 630$

pumps) equal to the total demand at the four destinations ($100+200+180+150= 630$ pumps) [8], [13].

A. Working Problem

The LP model can be solved by using the special structure of the constraint more conveniently using the transportation tableau shown below:

TABLE XIII
TRANSPORTATION COST

	Bangalore	Pune	New Delhi	Kolkata	Supply
Korea	75	70	85	80	80
Japan	86	82	96	90	150
UK	102	90	136	120	230
Lupton	100	98	115	112	170
Demand	100	200	180	150	

Using the **Robust’s Ranking Technique**, the solution for the above problem is obtained as follows:

Solutions:

Applying **BCM** for the above reduced problem the optimal solution is = \$ 10, 926.33,

Applying **VAM** method to the reduced problem the associated objective value is = \$ 10, 962.23,

Applying **North West corner** method to the reduced problem the associated objective value = \$ 12, 010.61,

Applying **Least cost method** to the reduced problem the associated objective value is = \$ 12, 211.77 [10], [17].

B. Finding the Optimal Solution for the Real Data in Triangular Fuzzy Numbers Working Problem:

Edwards – Vacuum Engineering Company manufactures vacuum pumps for the production of scientific instruments. The yearly demands for its product are 1890 pumps. The Company manufactures and distributes pumps once in four months.

The Pumps are distributed from four manufacturing centers to four distributions. The quarterly demands and supplies for a year are given below:

TABLE XIV
DEMANDS AND SUPPLIES FROM JANUARY TO APRIL

	Bangalore	Pune	New Delhi	Kolkata	Supply
Korea	71	66	80	78	70
Japan	82	76	92	84	130
UK	98	86	132	114	210
Lupton	96	92	110	100	140
Demand	90	180	160	120	

TABLE XV
DEMANDS AND SUPPLIES FROM MAY TO AUGUST

	Bangalore	Pune	New Delhi	Kolkata	Supply
Korea	75	70	85	80	80
Japan	86	82	96	90	150
UK	102	90	136	120	230
Lupton	100	98	115	112	170
Demand	100	200	180	150	

TABLE XVI

Applying **VAM** method to the reduced problem the associated objective value is = \$ 61, 010

Applying **North West corner** method to the reduced problem the associated objective value = \$ 12, 010.61,

Applying **BCM** for the above reduced problem the optimal solution is = \$ 10, 926.33,

Applying **VAM** method to the reduced problem the associated objective value is = \$ 10, 962.23,

Applying **North West corner** method to the reduced problem the associated objective value = \$ 12, 010.61,

Applying **Least cost method** to the reduced problem the associated objective value is = \$ 12, 211.77 [9], [17]. *C. Finding the Optimal Solution for the Real Data in Fuzzy Trapezoidal Numbers*

The above data can be taken as trapezoidal fuzzy numbers due to uncertainty. Therefore the Fuzzy TP can be formulated as follows.

TABLE XX
SUPPLY AND DEMAND IN TRAPEZOIDAL NUMBERS

	Bangalore	Pune	New Delhi	Kolkata	Supply
Korea	71,74,	66,68,	80,84,	78,81,	70,75,
	76,78	72,74	86,88	82,85	85,90
Japan	82,84,	76,80,	92,94,	84,86,	130,140,
	88,90	82,86	98,100	90,92	160,170
UK	98,100,	86,88,	132,134,	114,118,	210,220,
	104,106	92,94	138,140	122,126	240,250
Lupton	96,98,	92,94,	110,114,	100,112,	140,160,
	100,102	98,100	116,120	116,118	180,200
Demand	90,95,	180,190,	160,170,	120,140,	
	105,110	210,220	190,200	160,180	

TABLE XXI

TRANSPORTATION COST IN CRISP VALUE USING ROBUST RANKING

	Bangalore	Pune	New Delhi	Kolkata	Supply
Korea	74.5	70	84.5	81.5	80
Japan	86	81	96	88	150
UK	102	90	136	120	230
Lupton	99	96	115	111.5	170
Demand	100	200	180	150	

Solutions:

Applying **BCM** for the above reduced problem the optimal solution obtained is = \$ 59, 850,

DEMANDS AND SUPPLIES FROM SEPTEMBER TO DECEMBER

	Bangalore	Pune	New Delhi	Kolkata	Supply
Korea	78	74	88	85	90
Japan	90	86	100	92	170
UK	106	94	140	126	250
Lupton	102	100	120	118	200
Demand	110	220	200	180	

TABLE XVII

FROM THE ABOVE DATA WE CAN FORMULATE A TRIANG

	Bangalore	Pune	New Delhi
Korea	71,75,78	66,70,74	80,85,88
Japan	82,86,90	76,82,86	92,96,100
UK	98,102,106	86,90,94	132,136,140
Lupton	96,100,102	92,98,100	110,115,120
Demand	90,100,110	180,200,220	160,180,200

TABLE XVIII
SUPPLY AND DEMAND IN CRISP VALUE USING ROBUST RANKING

	Bangalore	Pune	New Delhi	Kolkata	Supply
Korea	74.5	70	84.5	80	80
Japan	86	82	96	90	150
UK	102	90	136	120	230
Lupton	99.5	97	115	110.5	170
Demand	100	200	180	150	
	D1	D2	D3	D4	Supply
S1	33	30.1	36.4	34.6	34.1
S2	37	35.1	41.3	38.5	64
S3	43.9	38.7	58.6	51.6	98.5
S4	42.9	41.9	49.5	47.6	72.2
Demand	42.8	85.6	76.9	63.5	

associated objective value is= \$ 66, 515

TABLE XIX

Solutions:

Applying **BCM** for the above reduced problem the optimal solution is = \$ 59, 850,
Applying **VAM** method to the reduced problem the associated objective value is = \$ 59, 950,

Applying **North West corner** method to the reduced problem the associated objective value is = \$ 63, 315,

Applying **Least cost method** to the reduced problem the associated objective value is = \$ 65, 210.

TABLE XXII
SUPPLY AND DEMAND IN CRISP VALUE USING CRT

	D1	D2	D3	D4	Supply
S1	29.1	27.2	32.9	31.7	31.1
S2	33.4	31.5	37.3	34.2	58.3
S3	39.7	35	52.9	46.7	89.5
S4	38.5	37.3	44.7	43.9	66.1
Demand	38.9	77.8	70	58.3	

D. Comparison Table

TABLE XXIII
COMPARISON OF ROBUST RANKING AND CRT

SI.NO	Fuzzy Numbers	Methods	Robust's ranking Technique	CRT
1	Triangular Fuzzy numbers	BCM	59, 905	10,936.33
		VAM	61,010	10,962.23
		NWCR	65, 275	12, 010.61
		LCM	66,515	12, 211.77
2	Trapezoidal Fuzzy numbers	BCM	59, 850	9,049.66
		VAM	59,950	10, 156.5
		NWCR	65, 315	10,072.43
		LCM	65, 210	9,690.87

III. CONCLUSION

The main objective of this TP is to determine the cost spent for shipping from one place to another so as to maintain the supply and demand requirements at the lowest transportation cost. The BCM can be used successfully to solve different problems of distribution of products that are commonly referred to TPs. Uncertainty in transportation cost brings imprecise data. Fuzzy numbers may represent this data. Ranking of fuzzy numbers are done using Robust ranking technique and CRT. Moreover fuzzy transportation cost and fuzzy optimal cost are more effective under the BCM.

For the comparative study we have used both the Robust Ranking Technique (RRT) and the CRT to Edwards Vacuum Company, Crawley, in West Sussex in the United Kingdom. We see that the transportation cost is reduced to minimum when we use the CRT under the BCM. We want to emphasize that the results obtained are approximate only.

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