Exploring Predictive Models for Student Success: A Multivariate Analysis of Further Mathematics Performance

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Abstract:

Academic success in further mathematics is perceived to have a significant contribution towards the development of science and technology in any nation. The main objective of this study is to identify the major determinants of students' academic success at WASSCE further mathematics. Two public schools in Hemang Lower Denkyira District constituted the study area with a sample of 84 students. Data for 2016/1017 WASSCE academic year group tracked from the documented records of the two selected schools in the District with 1:1 gender ratio were used for the study. Explanatory factors were; Mock examination, Age, Student Residence, School Location, and Gender on the target variable WASSCE grade. A three-stage stratified cluster sampling technique was used to select the two schools as well as classes and subject area under study. IBM SPSS Version 21 was used to analyse the data for the purpose to modelling multiple logistic regression with a binary response 'WASSCE grade' (upper grade/lower grade) against the systematic component of linear combination of predictor variables at 95% Confidence Interval. The study adopted the ex-post-facto research design. The model correctly classifies 77.4% of the overall cases indicating its prediction accuracy (robustness) and the extent to which it accurately predicted students' WASSCE grades. AUROC = 0.823, and Hosmer and Lemeshow test had p=0.540 > 0.05 indicating goodness-of-fit of the model. Three (mock examination, school location, and gender) out of five predictors made significant contribution to model with no multicolinearity among the predictors. These variables are the major determinants predicting students' WASSCE grade/performance in further mathematics. The study concluded that mock examination is reliable and thus should be centralized and supervised by the area education officer to make it more stringent, and there should be equity in resource allocation where both the less and well-endowed schools are equally supplied with teaching-learning resources, and finally, the education in its contents, designing and application should be gender solicitous.

Keywords: Academic Performance, Logistic Regression, At-Risk Students, Odds Ratio,

1. Introduction

The subject mathematics is perceived as having the most important role in growth of science and technology in the global world and thus makes it a central part of the world's culture. The subject is mostly experienced in our everyday life; thus, for the development of any nation, it must not undermine the relevance of Mathematics in her educational system.

Thus, if education is meant to prepare the youth for life to reflect in the field of science and technology, there is the need for students to study the subject, mathematics. These students then should give maximum attention and focus to pursuing the subject and must therefore demonstrate their fitness both in internal (mock) and external (WASSCE) examinations. Internal examinations as pre-WASSCE are selective, predictive and diagnostic in nature. They are supposed to reveal how successful teacher's instructions have been mastered by students. These examinations being predictive in nature can be used to selecting the students who will succeed in further academic endeavours. In most studies already reported, the two examinations show no significant correlation. Studies done on the relationship between mock and main exams in the Florida dental school reported no significant association between the two (Carole and Bates 2004).

Adeyemo, (2001) harangued that the major goal of the school is to work towards attainment of academic excellence by students. According to him, the school could have other outlying objectives but emphasis is mostly situated on the achievement of wide ranging studentship. Moreover, virtually anyone who is education minded places first-rate on academic achievement; where the expectation of parents is often excellent academic achievement of their progenies (Osiki, 2001). For that reason, the practice in which conducting of internal examinations before the main examination which aim at checking students' progress on academic programme is a common practice in many parts of the world but negative sentiments have always been raised from the culture of associating mock examinations with the main examinations. In most studies already reported, the two examinations show no significant correlation. Studies done on the relationship between mock and main exams in the Florida dental school reported no significant association between the two (Carole and Bates, 2004). While Alonge (1983) investigated the predictive validity of mock examination for WASSCE and concluded that mock mathematics examination helped significantly in predicting success in academic performance of students in WASSCE mathematics. The introduction of un-intended factors may construct irrelevant variance into the test scores and thus change the concept that the test intended to measure. As a result, the test scores obtained from the test may no longer provide an adequate basis for the kinds of inferences the test user intends to be able to make. This may bring differences and inaccuracies when comparing internal examinations and the final examinations (Ying and Sireci, 2007).

The American College Test (ACT) and the Scholastic Aptitude Test (SAT) are two standardized tests mostly used in U.S to assess academic ability a student possesses (Harachiewics *et al.,* 2002). It was suggested by Hoffman, (2002) that though ACT and SAT significantly predicted academic enactment, their scores as prior achievement, were not precise indicators. In contrast to this, Bunza *et al.* (1999) reported in his study that, highest scoring standards had a low correlation between internal and external examination while the lowest scoring standards had a high positive correlation between the mock and the final examination.

Several studies have been conducted based on the ordinary least squares (OLS) estimation. Spector, L.C., and Mazzeo, M. (1980) conducted the first study that applied a qualitative model to determine academic performance. Modelling students' performance might use different mathematical models such as Generalised Linear Models (GLM), Ordinary Least Square (OLS), and Standard Multiple Regression (SMR). The GLM is more advanced than others because of its capability of handling more multifaceted situations and analyses the simultaneous effects of multiple variables, including mixtures of categorical and numerical variables (Agresti, 2007, 2002; McCullagh & Nelder, 1989). The GLM is the extension of linear regression to study the phenomenon of a categorical response variable given continuous and/or categorical predictors following McCullagh and Nelder (1989). Applications abound in the fields of education (Aromolaran *et al.,* 2013), medicine (Sharareh *et al.,* 2010), and social sciences (Chuang, 1997). GLM is called by others as "nonlinear" because the mean of the response variable is often a nonlinear function of the covariates, but McCullagh and Nelder, (1989) consider it to be linear because the covariates affect the distribution of the response variable only through the linear combination of explanatory variables in a transformed form. The type of school and gender could be modelled using Generalized Linear Models investigated by Aromolaran *et al.* (2013) and Adejumo and Adetunji, (2013). Logistic regression has taken surpassing reputation in data modelling techniques for it does not seek nor rely on the assumptions of ordinary linear regression (Fadlalla, 2005).

Mock-Examination has been a popular concept for senior secondary school students preparing to sit for their senior secondary school certificate Examination (WASSCE). The importance of this examination (mock) has remained unpopular as the determinant of students' readiness for their final examination. The WAEC Chief examiners' report for (2006, 2012, 2015, and 2016) on mathematics stated categorically that students' performance fluctuates where on one hand, it turns to be encouraging and on the other hand, the performance is abysmally poor. Now, what could have brought about these variations? Could there be some major determinants which contribute to the performance of school candidates in WASSCE further/elective mathematics? If there are, then what is their predictive validity and to what extent do they contribute to students' WASSCE performance in further mathematics?
It is this that the present focused on exploring determinants that might have the main effect on students' academic performance at the WASSCE further mathematics.

1.1 *The need of Predicting Academic Performance of Students*

Ware and Galassi, (2006), and Veenstra *et al.,*(2008) indicated that models predicting students' academic performance may aid the instructor to rethink some proactive measures to favour the academically-at-risk students. Cooperative learning although found to inculcating an optimistic impact on educational success of students (Brush, 1997), studies revealed that ability-grouping could record greater academic success as compare to the mixed ability-grouping (Onwuegbuzie, *et al.,* 2003; Nihalani, *et al.,* 2010). Thus, with the predictive model, the instructor is allowed to identify the academic ability of a student.

1.2 Statement of the Problem

The perennial alarming rate at which students fail the WASSCE every year calls for attention and contemplation. The researcher investigated series of academic factors affecting students' performance. Some identified factors include poor instructional quality, students' residential status, and the location of the school (school environment–facilities in schools) as also previously identified by Oghuvbu (1998a, 2000b and 2003c). Students passed internal-examination in further mathematics but turn to fail the WASSCE further mathematics. The question is, "how valid and reliable is the content and standard of the internal examination questions for the students"? But the questions which readily come to mind are: what could be the major determinants that contribute to students' academic performance (grades) in further mathematics?, which model could best forecast students' academic performance in further mathematics in WASSCE, and to what extent (robustness) could this model predict their academic success? This is of concern. Consequently, the study sought to develop a multiple logistic regression model to identify the major determinants that predict student's WASSCE performance (grade), and also to identify students who are at-risk-academically at WASSCE further mathematics.

1.3 Research Objectives

The general objective of the study is to model logistic regression of the major determinants of students' academic performance on WASSCE grades in further mathematics.

1.3.1 Specific Objectives

1. To identify the determinants of students who obtain WASSCE upper grades in further mathematics.
2. To identify the predictive accuracy of the model.
3. To identify the determinants of academically at-risk students in further mathematics.

1.4 Research Questions

The study sought to address two research questions based on the research objectives of the study. These are:

1. What are the major determinants of students who obtain WASSCE upper grades in further mathematics?
2. How robust is the predictive model of the study?

1.4 Significance of the Study

Schools can rely on the findings to effect teaching and learning of Further Mathematics. District Directors and Heads of the selected schools can have source to intensify their request for better resources and material support for their schools. Also, Parents, students and other stakeholders in the educational enterprise can appreciate the problems encountered in the schools and be motivated to help the school administrations to yearn for quality education for their students.

3. Data and Methodology

3.1. Source of Data

This study relied on the academic performance and the Demographics of WASSCE further mathematics students for 2016/2017 academic year in Hemang Lower Denkyira District located on the north western part of the Central Region of Ghana. The district has a total of three (3) public and private senior high schools (SHS). There are two (2) publicly owned SHS and one (1) privately- owned SHS. Here, one of the public schools is sited at the district capital and the other at the non-district capital. The study covered the two public schools that offer Elective Mathematics as a subject of study.

As there was widely dispersed of the population in the district, the researcher adopted the rule known as *exclusion and inclusion* proposed by an International Standard of sampling following Gonzales et al (2008). The rule is such that, the selected schools from a given population of study will exempt other schools, the selected classes in each selected school for the study will exempt other classes, and the population of the selected subject area from each selected class will as well exempt other subject areas. The adoption of the rule made a plausible remark to confine the population of the current research to only the third year candidates who read further mathematics in the selected schools. This sampling procedure considered by TIMSS (2007) is generally referred to as a three-stage stratified cluster sampling technique. The current research had the sample size of 84 participants – 42 candidates each were from the two schools under study to make up the sample. The criteria used for the selection of these students included the fact that;

1. the participants must be students from the district that constituted the study area,
2. the schools must have presented the same students for both the school mock and the WASSCE in further mathematics for 2016/2017 academic year.

It is on these bases that the eighty-four students were selected.
The sex ratio and the residence ratio of the participant was 1:1– that is, 42 boys of which 21 were day students and the other 21 boarding students, and 42 girls of which 21 were day students and the other 21 boarding students. Since both the two schools are mixed schools. To end with, each participant from each selected schools was chosen by applying simple random sampling technique where a serial number was assigned to each student/participant based on gender. The randomly assigned serial number to each candidate was the bases of selected participants. The motive was thus to obtain a sample of bias-free.

3.2. Variables Information

3.2.1 Dependent/Response Variable

The response/outcome variable is a dichotomized variable "WASSCE grade" (upper grade/lower grade). The decision on this variable is based on its proximity to the concept of the student grade performance in further mathematics. According to Rootman et al., (2010), students' external examinations (WASSCE) grades in Further Mathematics where

WASSCE grade with A1 – C6 was coded as 1 and D7 – F9 coded as 0. Thus, in this study, students' WASSCE upper grade was coded as 1, and WASSCE lower grade as 0.

3.2.2: Explanatory/Predictor Variables

The prediction that either an event will or will not occur as well as identifying the major determinants (variables) to making such prediction is a plausible and a remarkable step in carrying out the research. The mock grades, school location, students' residence, and gender age of students are the independent variables used in the study. Table 3.1: Variable Information

VARIABLES		DATA TYPE	CATEGORICAL VALUES
Dependent Variable WASSCE grade	Upper grade Lower grade	Binary	1- A1—C6 D7— 0- F9
Independent Variables			
MCK- Mock examination		Binary	1- A1—C6 0- D7—F9
STR- Students residency		Binary	1- Day 0- Boarding
AGE- Age		Binary	1- 17—20 years 0- >20 years
SCL- School location		Binary	1- District capital 0- Non-district capital
GND- Gender		Binary	1- Male 0- Female

3.3 Research Design

The research used an ex-post facto research design also known as after-the-fact research, is a category of research design in which the investigation commences after the fact has occurred without the interference from the researcher. This was employed to identify major determinants which predict WASSCE grade of students and those who are academically-at-risk in WASSCE further mathematics. This design was employed because all the variables examined have already occurred and so they were collated from the existing school records for year 2017. Therefore, this did not require the manipulation of independent variables. Thus the researcher intends to identify the major determinants which can predict a response variable (WASSCE grade).

3.4 Methods of Data Analysis

3.4.1 Logistic Regression Model

Logistic regression which is a statistical technique for predicting the odds of an event given a set of explanatory variables is from the family of Generalized Linear Models

(GLMs), and it is modeled in Logit. Modelling in Logit is one of the methods used to analyse data where the response variable is nominal/categorical. It offers a relationship between the predictors and the log odds of the categorical dependent/response variable. However, when the categorical response variable is dichotomized to have exactly two responses (levels), it is termed as a binary response and the fitted model is binary logistic regression model. In the present study, the target variable *WASSCE grade* is a binary (upper grade = 1/lower grade = 0) response. Thus, data modelled a binary logistic regression. In logistic regression analysis, maximum likelihood estimation is employed after transforming the response variable (WASSCE grade) into a Logit variable and determines whether or not the natural logarithm of the odds of the response occurs. As such, the researchers employed the logistic regression to estimate the odds that a particular candidate scored WASSCE upper grade/lower grade in further mathematics.

According to Adjei, I. (2011), given the dataset with a binary response Y and a multiple explanatory variables which can be quantitative, qualitative (dummy variables) or both, the Multiple Logistic Regression model can be fitted in log odds as

$$\text{logit}(\pi) = \text{logit}\left(\frac{\pi}{1-\pi}\right) = \text{logit}[P(Y=1)] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n$$

(1) for $n > 2$ or equivalently

$$\pi(x) = \frac{\exp(\beta_0+\beta_1x_1+\beta_2x_2+\cdots+\beta_nx_n)}{1+\exp(\beta_0+\beta_1x_1+\beta_2x_2+\cdots+\beta_nx_n)} \quad (2)$$

Where $\pi = \Pr(Y = 1)$ the probability of the student having an upper grade in further mathematics,

β_0 = constant, $x_1, x_2, x_3 \ldots, x_n$ = explanatory variables, and

$\beta_1, \beta_2, \beta_3 \ldots, \beta_n$ = coefficients.

The estimate β_i indicates the effect of x_i on the logarithm odds that $Y = 1$, holding the other $x's$ constant. Example, $\exp(\beta_i)$ is the multiplicative effect on the odds of a 1-unit increase in x_i, at fixed levels of the other $x's$.

Multiple logistic regression model has three components:

- Random Component :- WASSCE grade (Y) – logit[E(Y)]
- Systematic Component: -
  - Gender (GND = x_1)
  - School location (SCL = x_2)
  - Mock examination (MCK = x_3)
  - Students' residence (STR = x_4)
  - Students' age (AGE = x_5)

$$\beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_5x_5$$

If logit [E(Y)] = logit ($\pi(x)$),

- Link Function:- $\eta = \text{logit}(\pi(x)) = \text{logit}\left(\frac{\pi(x)}{1-\pi(x)}\right)$

Hence, the model:

$\pi(x)$

$$\text{logit}\left(\frac{\pi(x)}{1-\pi(x)}\right) = \text{logit}[(Y=1)] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5$$

(3) equivalently;

$$\pi(x) = \frac{\exp(\beta_0+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3+ \beta_4 x_4+\beta_5 x_5)}{1+\exp(\beta_0+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3+ \beta_4 x_4+\beta_5 x_5)} \quad (4)$$

3.4.2 Odds Interpretation

The understanding interpretation of logit model is facilitated when it is explained in terms of odds and odds ratio values. Odds is the ratio of the probability of success to the probability of failure. Odds of an event is the ratio of the probability of the event's success to the probability of the event's failure. Thus, odds ratio is the ratio of the two odds. Odds specify the extent to which the likelihood an event may occur against the event might not occur. The odds ratio for an explanatory variable is explained in relation to a decrease or increase. Odds lie within the interval [0, 1] inclusive. Ratio of odds more than zero for a given explanatory variable is a multiplicative effect by how much the odds of the results will increase. Likewise, ratio of odds less than zero for a given explanatory variable is a multiplicative effect by how much the odds of the result decrease. Odds may be expressed as $p/(1-p)$, where p is the probability that an event will occur and $(1-p)$ is the probability that an event will not occur. As a result, odds ratio is the odds of an occurring event to the odds of an event not occurring. Odds Ratio $= e^{\beta_1}$ is the multiplicative effect, where β_1 is the unknown parameter.

3.5: Assessing the Accuracy of the Model/Goodness of Fit

"*Goodness of Fit and the Deviance* to detect lack of fit searches for any way the model fails. A goodness-of-fit test compares the model fit with the data. This approach regards the data as representing the fit of the most complex model possible – the saturated model, which has a separate parameter for each observation. Denote the working model by M and the saturated model by S. In testing the fit of M, we test whether all parameters that are in the saturated model but not in M equal zero (Adjei, I. A., 2011)." That is;

H_0: *model M holds* vs H_1: *saturated model*

Likelihood- Ratio test statistic: $-2(L_M - L_S)$ = Difference in deviances.

Some of the terms (predictor variables) in the model might not be needed; therefore, they must be removed. What term is needed in the model? To do this, a test is conducted where maximized values of logarithm-likelihood for such a model are compared with the simpler model containing no such terms. The comparison of the maximized logarithm-likelihood (L_1) of the complex model with the maximized logarithmlikelihood (L_0) of the simpler model where such parameters equal to 0 is what the likelihood-ratio test does (Adjei, I. A., 2011). Any particular variable/term in a logistic regression model that proves significance is tested by the hypothesis;

$H_0: \beta_i = 0$ vs $H_1: \beta_i \neq 0$.

Whether or not a particular term (predictor variable) is statistically significant can be determined using two methods. That is; the difference between the deviances of the two models where one contains the predictor and the other is without the predictor is one of the methods used to check the significance of a particular predictor.

G = [Deviance of model without predictor – Deviance of the model with predictor]

3.5.1: Classification Table

The accuracy of the predictive model is also examined using the criteria; *"Accuracy Rate (AR)"* and *"Misclassification Rate (MR)."* The predictive accuracy of the logistic regression model (i.e, an accurate prediction) in the present study was well-explained as a situation where the value predicted falls in the interval 70-100% given the observed value. And here, the error of prediction is ($\pm$10%). A high percentage of accuracy rate of predictions implies the goodness of the model.

3.5.2: AUC – ROC Curve

When it comes to a classification problem, an AUC - ROC Curve can be used. When we need to check or visualize the performance of the multi - class classification problem, AUC (Area Under the Curve) - ROC (Receiver Operating Characteristic) curve is used. It is one of the most important evaluation metrics for checking any classification model's performance. AUC - ROC curve is a performance measurement for classification problem at various thresholds settings τ which is mostly 0.05. ROC is a probability curve and AUC represents degree or measure of separability. It tells the degree to which the model is capable of distinguishing among/between explanatory variables. The higher the AUC, the better the model is at predicting 0s as 0s and 1s as 1s. Analogously, the higher the AUC, the better the model is at distinguishing between students scoring WASSCE upper grades and those scoring WASSCE lower grades. In one case, a good model has AUC close to the 1 which means it has a good measure of separation between the two groups while a poor model has AUC close to the 0 which means it has poor measure of separability. In another case, a perfect or excellent model has AUC equals 1(i.e. no overlapping of the distribution) while AUC which is approximately 0, means the model is actually reciprocating the outcomes. That is, the model is predicting negative outcome as a positive outcome and vice versa (i.e. it is predicting 0s as 1s and 1s as 0s). Now, when AUC is 0.5, it means the model has no class separation/discrimination ability between the two levels of the response variable.

According to Liao & Triantaphyllou, (2008), sensitivity (the proportional measure of samples with *"Yes"* that are correctly classified regarding a given cut-off value) and specificity (the proportional measure of samples with *"No"* that are correctly classified regarding a given cut-off value) of the model given a threshold/cut-off value are also displayed by the ROC curve. Also, the probability of a situation where a statistic falsely forecasts the non-prevailing situation regarding the cut-off value is what 1 – Specificity measures. To illustrate this, label WASSCE grade as y ∈ {1- upper grade, 0 – lower grade} and be given a cut-off or threshold value, τ. The observation classifies correctly as 1upper grade should result be greater than τ, and as 0-lower grade should outcome be is less than τ. The outcomes of this can be shown in a confusion matrix table. See table 3.1 below.

Table 3.1 ROC Table

	Predicted WASSCE Grade	
Observed WASSCE Grade	Upper grade	Lower grade
Upper grade	TP	FN
Lower grade	FP	TN

TP = True-Positive, FP = False-Positive, FN= False-Negative, and TN= True-Negative
Sensitivity = (TP) / (TP + FN)
Specificity = (TN) / (FP + TN)

4. **Results**

Table 4.1: Categorical Variable Information

			N	Percent
Dependent Variable	WASSCE grade	0	46	54.8%
		1	38	45.2%
		Total	84	100%
Factors	MCK	0	33	39.3%
		1	51	60.7%
		Total	84	100%
	AGE	0	41	48.8%
		1	43	51.2%
		Total	84	100%
	STR	0	42	50.0%
		1	42	50.0%
		Total	84	100%
	SCL	0	42	50.0%
		1	42	50.%
		Total	84	100%
	GND	0	42	50.0%
		1	42	50.0%
		Total	84	100%

Table 4.1 above shows the information on percentage of the participant in the study. 84 sampled students who sat for WASSCE indicated that 54.8% had lower grades (D7 – F9) while 45.2% upper grades (A1 – C6) at WASSCE further mathematics. 39.3% had lower grades in mock examination while 60.7% upper grades. 51.2% of the participant had the age between 17 – 20 years while 48.8% were above 20 years. 50% each were day and boarding students. Students from the district and non-district capital were both 50% each. 50% each were male and female students.

Table 4.2 Correlation

		WASSCE	MCK	AGE	STR	SCL	GND
WASSCE	Pearson cor.	1	0.388	0.122	-0.144	0.335	0.144
	Sig. (1-tailed)		0.000	0.135	0.096	0.001	0.096
	N	84	84	84	84	84	84
MCK	Pearson cor.	0.388	1	-0.005	-0.073	0.122	-0.122
	Sig (1-tailed)	0.000		0.481	0.254	0.135	0.135
	N	84	84	84	84	84	84
AGE	Pearson cor.	0.122	-0.005	1	-0.167	0.262	-0.167
	Sig (1-tailed)	0.135	0.481		0.065	0.008	0.065
	N	84	84	84	84	84	84
STR	Pearson cor.	-0.144	-0.073	-0.167	1	-0.333	0.143
	Sig. (1-tailed)	0.096	0.254	0.065		0.001	0.096
	N	84	84	84	84	84	84
SCL	Pearson cor.	0.335	0.122	0.262	-0.333	1	-0.286
	Sig. (1-tailed)	0.001	0.135	0.008	0.001		0.004
	N	84	84	84	84	84	84
GND	Pearson cor.	0.144	-0.122	-0.167	0.143	-0.286	1
	Sig (1-tailed)	0.096	0.135	0.065	0.097	0.004	
	N	84	84	84	84	84	84

Correlation is significant the at 0.01 level (1 - tailed)

In table 4.2 above, it can be observed that the correlations between the five variables (MCK, AGE, STR, SCL, and GND) is small, less than 0.8 which does not give the researcher any indication of possibility of collinearity. Since the variables are not highly correlated, the researcher can go ahead to interpret the other outputs. Also, there exist a positive correlation between the WASSCE grade and the other four input factors (mock examination, age, school location, and gender), but there is a negative correlation between WASSCE grade/performance and one input factor (students' residence). There is an approximately a moderate correlation between WASSCE performance and the school mock performance supporting the study carried out by Hicks and Richardson (1984), where they recorded a moderate correlation between student's diagnostic score and the overall GPA.

4.1: Goodness of Fit of the Model Assessment

Fitting a model to a dataset is a very essential where it is enquired about the degree to which the fitted values of the outcome/predicted variable under the model in question compare with the observed values. The model is good is and accepted if the observed values agree with corresponding fitted values. Otherwise, it is a poor model and should be rejected so that the model is revised. This aspect of model performance or model adequacy is what is termed as goodness-of-fit. In this study, the overall significance of logistic regression model is assessed using; Likelihood Ratio Test, Classification Table, Omnibus test of the model coefficients, Hosmer and Lemeshow goodness-of-fit test, and AUC-ROC Curve. The present study made use of Statistical Package of Social (IBM SPSS Statistics) version 21.

4.1.1: Classification Table

The logistic regression model was characterized as useful using the benchmark of 25% improvement over the rate of accuracy achievable by chance alone following (*SW388R7 Data Analysis & Computers II*). The independent/explanatory variables even though might have no association to the defined groups by the response variable; the researcher would still expect to be correct in the predictions. This explains the bychance-accuracy. In table 4.13 is a Step 0 indicates the number of cases in each group (independent variables not yet included). The proportionality of cases in the largest group equals total/overall percentage (55%). The proportionality by-chance-accuracy rate was estimated by calculating the square of sum of the proportion of cases for each group based on the number of cases in each group in the classification table at Step 0, ($0.55^2 + 0.45^2 = 0.505 = 51\%$). The proportionality by-chance-accuracy criteria is 63.75% (i.e. 1.25 x 51% = 63.75%).

Table 4.3: Classification Table

Observed			Predicted		
	WASSCE			Percentage Correct	
			0 46	.	
Step 0	WASSCE	0		0	100.0
		1	38	0	0.0
	Overall Percentage				54.8

a. Constant is included in the model. b. The cut value is .500

The comparison between the actual scores and the predicted scores is tabulated in table 4.4. It shows the number of cases in each group at Step 1 (independent variables after included).

Table 4.[1]: Classification Table

Observed			Predicted		
			WASSCE		Percentage Correct
			0	1	
Step 1	WASSCE	0	31	15	67.4
		1	4	34	89.5
	Overall Percentage				77.4

a. The cut value is .500

Concerning the predictive power/ability of the model, the response variable is assigned "1", if the student received upper grade and "0" if he/she scored a lower grade in the subject of study. It can be observed in table 4.4. that, 31 students were predicted to score WASSCE lower grade by the model with the five predictors and actually scored lower grade, whereas 4 were predicted to score lower grade and in fact scored upper grade. Likewise, it was predicted that, 34 students will obtain WASSCE upper grade and indeed they obtained upper grade, however it was predicted also that, 15 students will obtain WASSCE upper grade but scored lower grade. The Overall Percentage of 77.4% is calculated as the ratio of the summation of the diagonal values to the total number of cases. The diagonal values represent the cases that are correctly classified (classification accuracy rate = 77.4%) and the off-diagonal values indicate cases which are misclassified (misclassification rate = 22.6%). The accuracy rate indicates the extent to which the researcher's predictions show accuracy. Although, this might not be excellent, it revealed a strong evidence of performance ability beyond the reference model in which the predictions accuracy were 54.8%.

Table 4.5: Omnibus Tests of Model Coefficients

	Chi – square	df	Sig.

[1] ***.1.2: Omnibus Tests of Model Coefficients***

The association between combination of predictor variables and the predicted variable is statistically built depending on the model's significance given the chi-square at step 1 once the explanatory variables have been included in the model analysis. The indication in table 4.5 shows whether or not the model with the predictor variables best fits the data than the baseline model. Thus, it offers the researcher better prediction of students' academic performance.

Step 1	Step	32.663	5	0.000
	Block	32.663	5	0.000
	Model	32.663	5	0.000

The significance can be found in the last column of table 4.4. The present analysis showed that the odds of the model chi-square (32.663) as < 0.001 was less than or equal to the level of significance 0.05. The presence of an association between mock, age, gender, student residence, and school location as explanatory variables in one case and WASSCE grade as the predicted variable in second case was supported. This explained that the model with these five explanatory variables at step 1 fits well compare to the model having no such explanatory variables.

4.1.3: Hosmer and Lemeshow Test

Hosmer and Lemeshow Test also show the fitness of the regression model.

Table 4.6: Hosmer and Lemeshow Test

Step 1	Chi - square	df	Sig.
	6.968	8	0.540

It can be seen in Table 4.6 that the model fit is acceptable χ^2 (9) = 6.968, p = 0.540, and shows that the observed values were not significantly different from the values predicted by the model. The p-value in table 4.6 must be greater than the established cut-off value which is mostly 0.05 to show that there exists no difference between the observed values and the predicted values and thus, prove goodness-of-fit of a model.

4.1.4: AUC-ROC Curve

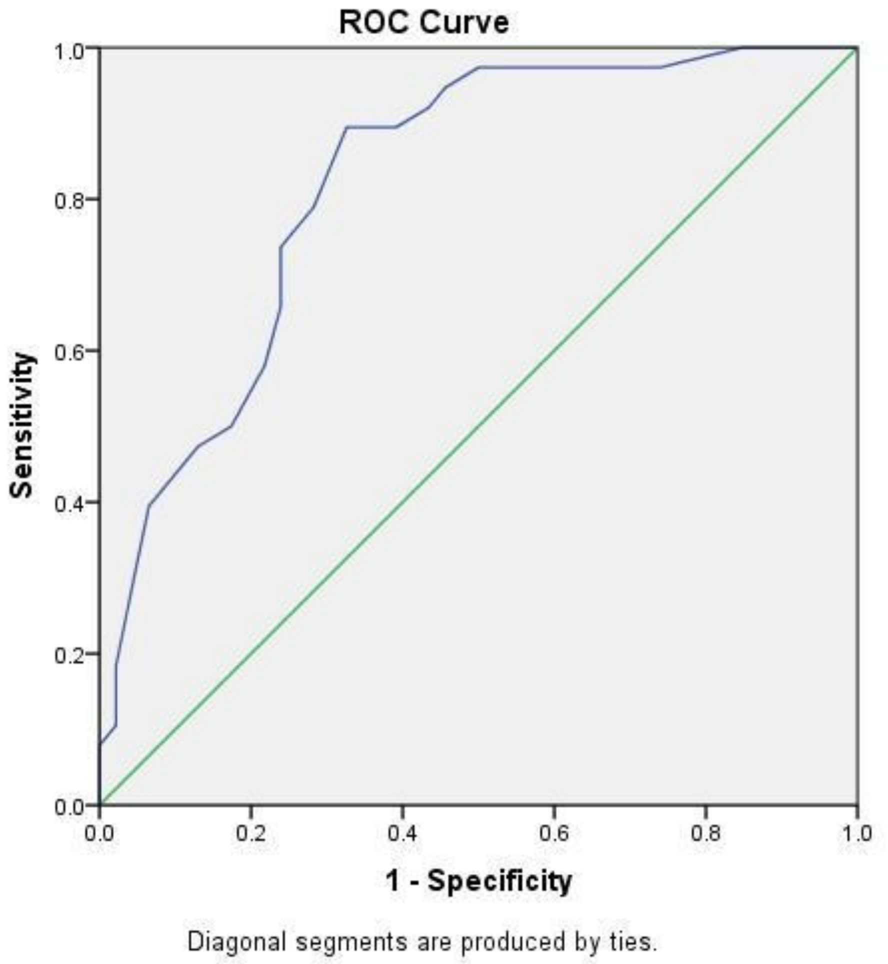

Figure 4.1 illustrates the ROC curve where the true positive (sensitivity) is labelled along vertical axis and false positive (1-Specificity) plotted along the horizontal axis. Any point plotted on the curve relates to a specific threshold or cut-off value. Any point on the curve considered perfectly is within the interval/range [0, 1], which ascertains that all variables which are statically significant are correctly classified and those that are not statistically significant are misclassified.

The curve coinciding with the vertical axis indicates the significance of the model. Thus, the model is significant.

Table 4.8: Area Under Curve (AUC)

Test Result Variable(s): Predicted probability

Area	Std. Error	Asymptotic Sig.	Asymptotic 95% Confidence Interval	
			Lower Bound	Upper Bound

			.734	.911
.823	.045	.000		

The test result variable(s): Predicted probability has at least one tie between the positive actual state group and the negative actual state group. Statistics may be biased.
a. Under the nonparametric assumption
b. Null hypothesis: true area = 0.5

In Table 4.8, the Area Under the Curve estimated is 0.823 at 95% confidence interval (0.734, 0.911). The AUC with p-value = .000 is 0.823 which is significantly different from 0.5. This proves that the logistic regression model significantly classifies cases correctly and that it is better than by chance model. Since the AUC is greater than 0.5 and close to 1, the model specifies good classification accuracy. This again proves the goodness-offit of logistic regression model.

4.1.5: Model Summary

In table 4.7, two measures were shown by the would-be R-Square statistics being Cox & Snell and Nagelkerke. Though both the measures are equally valid, they somewhat adopt different formula. As such, Cox & Snell is 0.322, and Nagelkerke is 0.431.

Table 4.7: Model Summary

Step 1	-2 Log likelihood	Cox & Snell R-Square	Negelkerke R-Square
	83.023	0.322	0.431

a. Estimation terminated at iteration number 5 because parameter estimates changed by less than .001.

4.3: Variables in the equation

									Upper
Step 1	MCK	-2.295	0.644	12.698	1	0.000	0.101	0.029	0.356
	AGE	-0.442	0.561	0.620	1	0.431	0.643	0.214	1.931
	STR	0.231	0.575	0.161	1	0.688	1.259	0.408	3.885
	SCL	-2.083	0.678	9.427	1	0.002	0.125	0.033	0.471
	GND	-1.997	0.685	8.508	1	0.004	0.136	0.036	0.519
	Constant	2.662	0.828	10.340	1	0.001	14.326		
Step 2	MCK	-2.287	0.640	12.760	1	0.000	0.102	0.029	0.356
	AGE	-0.462	0.560	0.680	1	0.410	0.630	0.210	1.888
	SCL	-2.144	0.665	10.380	1	0.001	0.117	0.032	0.432
	GND	-1.975	0.680	8.430	1	0.004	0.139	0.037	0.526
	Constant	2.805	0.752	13.923	1	0.000	16.540		

		B	S.E.	Wald	df	Sig.	EXP(B)	95% C.I. for EXP(B) Lower	Upper
Step 3	MCK	-2.267	0.641	12.517	1	0.000	0.104	0.030	0.364
	SCL	-2.235	0.656	11.612	1	0.001	0.107	0.030	0.387
	GND	-1.930	0.676	8.162	1	0.004	0.145	0.039	0.546
	Constant	2.609	0.705	13.672	1	0.000	13.580		

Table 4.5: Variables in the Equation

a. Variable(s) entered on step 1: MCK, AGE, STR, SCL, GND.

Outputs from research question one was used to fit logistic regression model that predict student's academic performance in terms of grade at WASSCE in further mathematics.

Table 4.5 above shows the Backward Stepwise (Wald) method. Those variables that are significant and those that are not can be seen. The variable is statistically significant if its coefficient has significance less than 0.05. Otherwise, it is not. Thus table 4.5 above is a backward stepwise (Wald) method and indicates that, the odds of the Wald statistic at 0.05 level of significance or at 95% confidence interval for the predictors; mock examination, school location, and gender are < 0.001, 0.001 and 0.004 respectively are less than or equal to the 0.05 level of significance. The results in this table show that only three out of five predictors proved to have the regression coefficient β as significant. They include; Mock examination, School Location, and Gender. These predictors contributed to the predictive accuracy of the logistic model.

Independent variable, AGE has a significance of 0.431 and 0.410 in step 1 and step 2 respectively therefore; it is not statistically significant if students are of age below or above 20 years in assessing their WASSCE performance in further mathematics. Likewise, the student residence (STR) variable with a significance of 0.688 does not prove to be statistically significant. Therefore; it is also not statistically significant if students school as day or boarding students when it comes to assessing their academic performance at WASSCE further mathematics in terms of grades. These variables therefore should not be included in the model.

In Table 4.5, column seven shows the estimated odds ratios, e^{β}. The range of estimates is from 0.104 to 0.145. The value 0.145 being the largest was obtained for the gender variable, while those for Mock and School Location each approximately are 0.10 and 0.11 respectively. These estimates are not that quite high but still indicate that with quite high grade in Mock, school location in the district capital and the believe that difference in gender performance in the subject exists, not academically-at-risk students scoring WASSCE upper grade in the subject under study have higher odds ratio than those academically at-risk. The insignificant association between WASSCE grade and the predictors; students' residence and age in table 4.5 may be as a result that both day and boarding students at various ages in the Hemang Lower Denkyira District plan and use the same time management in their studies. The application of correlation analysis as shown in table 4.2 above was noted that no significant correlation existed between the predictors included in the model. From table 4.5, the model relating students' WASSCE grade, and gender, school location, and mock examination is fitted in Logit form as given below; $\text{logit}[\hat{\pi}(x)] = \log\left(\frac{\hat{\pi}(x)}{1-\hat{\pi}(x)}\right) = 2.609 - 1.930x_1 - 2.235x_2 - 2.267x_3$ (5) where

$$\hat{\pi}(x) = \frac{e^{(2.609-1.930x1-2.235x2-2.267x3)}}{1+e^{(2.609-1.930x1-2.235x2-2.267x3)}} \quad (6)$$ and

x_1 = Gender
x_2 = School Location
x_3 = Mock Examination

In table 4.5, Age (AGE) and student residence (STR) were not included in the model since they are found not statistically significant. The values of β_1, β_2 and β_3 in the model are negative which mean a unit increase in each one of them holding the other two constant will decrease the odds/probability of scoring a lower grade on the WASSCE grade variable.

5. Discussion

Considering Exp (B) in Table 4.5, it can be said that the gender has an impact on the model; male performance is 0.145 times higher than female performance. In other words, Wald for parameter GND is 8.16 which show that when other parameters are unchanged, the chances for a higher performance among male are 0.145 times better than among females.

Also, Wald for parameter SCL, 11.612, it shows that, controlling for other parameters, the chances for a higher performance among students schooling in district capital school(s) is 0.107 times better than the performance among students who consider schooling in non-district capital school(s).

Wald for parameter MCK is 12.517 which show that holding other parameters constant, the chances for higher performance among students who obtain upper grade in mock exams is 0.104 times better than those who might obtain lower grades in mock exams.

5.1: Odds Ratio Interpretation

In terms of odds by considering the Table 4.5 , it can be interpreted as; controlling the effect for school location and gender, estimated odds of WASSCE grade for an upper grade in mock is 0.104 times estimated odds for a lower grade. In the same way, controlling for school location and mock examination, estimated odds of WASSCE grades for males is 0.145 times estimated odds for females. Again, controlling for gender and mock examination, estimated odds of WASSCE grade for students in district capital schools is 0.107 times estimated odds for students in the non-district capital schools. Consequently, the value of Exp(B) in that same table for mock is 0.104 which implies that a unit increase in obtaining an upper grade in mock examination would decrease the odds that a student would obtain lower grade in further mathematics in WASSCE by 0.104 more times. Likewise, the incident of school location is 0.107 which implies that a unit increase in student' rate of schooling in urban or district capital would decrease the odds of a student to score lower grade in further mathematics in WASSCE by 0.107 more times, holding other variables constant.

It is significantly therefore, to say that independent variables that are significant in the model by all the various methods used as indicated above were MCK, SCL, and GND, respectively, mock examination, school location, and gender.

The research objective three identified the most important determinants of students who were not able not obtain WASSCE upper grade that is between A1 and C6 inclusive related students' dataset. Outputs from research question one was used to identify the determinants of SHS candidates who could not obtain the WASSCE upper grade. Students who did not obtain the WASSCE grade between A1 and C6 were academicallyat-risk. The students' data from the school was exploited to find the variables contributing as the impediment on students' academic performance in the subject (further mathematics). The decision/classification tree model revealed that mock examination, school location, and gender were the most significant variables. To identify the category of students who need a speedy attention at the start of reading the subject, these significant variables were further exploited.

6. Conclusion and Recommendation

This research was undertaken to identify the major determinants of students' academic performance at WASSCE further mathematics subject, where the response variable was WASSCE grade and the predictor variables are; school mock, school location, students' residence, gender, and students' age. The Pearson correlation used to analyze the results showed that WASSCE grade is positively correlated to the four factors, but negatively correlated to one factor. There is also a low correlation between factors. This thus revealed that there exists no multicollinearity among the factors of WASSCE grade. The logistic regression model identified variables (mock examination, school location, and gender) that proved potent at predicting academic performance in further mathematics in senior high school certificate examinations to an appreciable extent with mock results having the highest predictive ability. The research finding (mock) supported the claim of Alonge (1983) who investigated the predictive validity of mock examination for WASSCE and concluded that mock mathematics examination helped significantly in predicting success in academic performance of students in WASSCE mathematics. These significant variables are the major determinants contributing to the student's success in further mathematics at WASSCE. Student who is deficient in mock examination performance, and acquire his/her senior high school education without considering the location of the school might be academically at-risk at WASSCE further mathematics.

Regarding the predictive ability/power (robustness) of the regression model, HosmerLemeshow test proved that the model is a good fit. That is, the model correctly classified 77.4% of the overall cases; again indicating the robustness of or how robust is the predictive model. Also, the AUC-ROC Curve (AUROC) was found to be 0.823 and indicates the goodness-of-fit of the logistic regression model.

The following recommendations based on the finding of the present research.

1. Mock examination should undergo the process of standardization to be able to compete favourably with WAEC, which is the standardized examination.
2. School counselling for academically at risk students.
3. Education in its contents, designing and application should be gender solicitous to correct current incongruities in our culture and education. In essence, equality of gender planning, sensitivity, training and education is needed.

4. Schools in rural areas should be endowed with quality teaching-learning resources as schools in the urban centres.
5. Lastly, Analysis of other Generalized Linear Models (GLMs) by considering more variables influencing students' academic performance in further mathematics or other related subjects should be studied. And thus such research work may undertake the validation of logistic regression models.

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