

Enhancing Mathematics Education through Visualization: A Conceptual Framework for Engineering Disciplines

Nóra Gábor Kovács, Erszébet Bálintné Pál

Nóra Gábor Kovács, University of Szeged, Szeged, Hungary

ABSTRACT

Students in engineering education need tools to gain insight into the ever-increasing complexity of engineering problems and possible solutions in the 21st century (e.g. seeking the reasons for the recent bridge-collapse in Genova). One of these tools could be the utilization of mathematical knowledge and skills – but many engineering students are undermotivated in studying mathematics.

Not only Comenius but our digital age also prefers visualization over textual comprehension, as the Net generation is visually literate. Newer interdisciplinary research findings in brain functions and brain maturation are worth to be integrated into the pedagogy of teaching mathematics to engineers.

Methodologically, in order to improve the quality of teaching Mathematics in engineering education at a Hungarian university, both findings in brain-research as well as theories of adult learning have been analysed from the perspective of visualization. The other direction of the work was focused on different types of visualization in Mathematics (according to Guzman), particularly in textbooks for engineering students. Ten textbooks, (among them the newly developed

„Mathematics 1” at the Széchenyi István University), available both in print and online in Hungary have been compared from visual aspects. The current Curriculum of the subject „Mathematics 1” has also been analyzed from visual aspects. Findings show the need for a wider variety of visualization.

Systematically detailing all of the above-mentioned perspectives and findings of dataprocessing contribute to developing an up-to-date conceptual framework for improving the quality of teaching Mathematics in engineering education at a Hungarian university, and it might be useful for other universities as well.

1 INTRODUCTION

Experts of many engineering disciplines were shocked by the news on the recent bridge-collapse in Genova. A highly unlikely interrelationship of very complex ageing processes in an artificial, human creation – a large scale mechanical structure – along with rare weather circumstances (processes of nature) led to the appalling catastrophe. It was a real challenge for engineers to find the reasons for that surprising event, and to elaborate solutions to avoid similar crashes in the future.

It would be impossible to gain insight into the ever-increasing complexity of engineering problems and possible solutions in the 21st century without utilizing the tools of Mathematics. This paper wishes to contribute to the development of the quality of teaching Mathematics in engineering education by shedding light on the ever-growing importance of visualization in our digital age.

2 CONTEXT OF THE PROBLEM AND AIMS OF THE ANALYSIS

According to the predictions of the World Economic Forum (2019) [1], *Complex Problem Solving, Critical Thinking and Creativity* are leading the list of *Top 10 Skills*, needed by the workforce on the labour market in the future. New elements among the Top 10 Skills are *Emotional Intelligence* and *Cognitive Flexibility*. Mathematics is one of the subjects of engineering education, which offers an opportunity for developing all of the above-mentioned skills. However, teaching methods and the context of the tasks need to be revised and updated continuously. It is expected by society that the theoretical knowledge along with the practical skills of engineers should be adaptable and still produce high quality solutions in different context as well (see United Nations' Sustainable Development Goals, 2015) [2]. That is why engineering education itself needs to reflect these expectations and teach global and adaptable guidelines.

In spite of these ideal goals, many engineering students are unsuccessful and undermotivated in mathematics, do not pass the exam in Mathematics and it is a dreaded or dull subject for them. That is why we decided to take steps in order to reduce the proportion of these students in engineering education.

The Net generation prefers visualization over textual comprehension [3], so our work targeted to analyse the inner nature of learning and motivation of engineering students on the basis of adult learning theories and findings of brain research from the perspective of visualization. Our other goal was to investigate whether the learning materials available in Hungary for the students along with the Curriculum of the subject *Mathematics 1* take the specialities of visual learning into consideration.

3 THEORETICAL BACKGROUND

3.1 Adult learning

According to Dewey's classical learning theory, learning takes place when one is doubtful about how to go on, how to act in the future [4]. That disorienting dilemma, or disjuncture is well-known at the beginning of solving a mathematical problem, and appear as a starting point in the theories of Jack Mezirow [5] and Peter Jarvis [6] as well. Mezirow thinks that to be able to understand and solve the problem (e.g. a mathematical problem), a *transformation* in the adult's personality is necessary. The basis of the positive progress of a person is the development of a critical change of perspective. This process can be assisted through social interactions, by learning from each other, - and by visualization.

Both of the transformative learning theories of Mezirow and Peter Jarvis's concept lie on a *constructivist* basis [7,8]. The aim of constructivistic teaching is to widen the repertory of possibilities in thinking and acting. Learning is not determined by teaching, but rather by the cognitional and emotional structures of the learner's mind. These structures are connected to biographical experiences, in the particular case of Mathematics they may be linked to preconceptions (prejudices) about the subject's learnability and to previous experiences in learning it.

Jarvis uses the term "Life-world" defined by the German Horst Siebert as a construction or a meaning schema which "is built up of our knowledge, beliefs, interpretation patterns, permanent themes and includes our action plans, ... and our forgotten or repressed contents of conscience" [9]. We think that these contents are visualized constructions. The sustainability or unsustainability of a construction

depends on its viability (in constructivist terms). From the student's perspective, a new knowledge is viable if it suits him/her and helps him/her achieve his/her goals. That is why it is crucial to make the goal of studying mathematics for the engineering students clear. The viability of a construction may cease in time. For instance, a construction of solving a mathematical problem may become destructive, meaning that it does not help to pass the exam, and as a result, it has to be reconstructed.

Informing or communicating in a learning community on mathematical problems means offering an opportunity for the participants to make connections (that is compatibility) between different constructions on solving a problem. In other words: offering the opportunity for them to build more structured, more complex and better informed „mind maps” than they had before. With the broadening of metacognitive (reflective) knowledge [10], the necessity of conceptual change may become more apparent. Metacognition supports and makes self-directed learning easier, which is the precondition of lifelong learning.

Similarly to Mezirow, Jarvis also emphasises the transformative characteristics of learning, particularly the transformation of people through new experiences. How people interpret their experiences (see social constructivism) depends on the social and cultural context as well [11]. Perception and the physical, biological processes are significant in learning. That means that the perceptions of figures and an inner visualization of the problem may help or hinder the learning process. According to Jarvis, the essence of learning is that the initial feeling of absence or confusion transforms into knowledge, competence, attitude, value, emotion, etc. This transformation can occur by thinking, by taking actions, in an emotional way or in any combination of all three of these. Emotions have a significant effect on our way of thinking, motivations, attitudes and values as well. As in the course of learning meaning schemas are being transformed, in this case the visualization of meaning schemas can support the process of transformation. Different types of visualization can approach challenging problems and/or solutions in new ways.

Comenius was the first in the history of didactics, who highlighted the role of sensation in learning and teaching in his *Didactica Magna* published in 1657, and in his illustrated textbook *Orbis Pictus* (1658). Our digital age [12] also prefers visualization, as the Net generation is visually literate. Many youngsters use the internet and their mobile-phone to seek challenging visual experience instead of reading texts [13].

Newer investigations in the motivational profiles of adult learners [14] show that some of the adults with lower levels of academic self-concept and lower use of deep-learning strategies, (who would also supposedly have difficulties in learning mathematics,) might have experienced academic failure(s) with negative images in their life history.

Brain researches pay attention to a special process in the brain called myelination which starts before birth but ends until late adolescence. In this process myelin sheath is not fully formed until around ages 24-26 which causes that adolescence and young adults learn differently. Consequently in these ages visual representations can contribute to a wider and deeper understanding [15].

3.2 Typology and other approaches of visualization in mathematics

Miguel de Guzman, a Spanish mathematician created a typology for visualization in Mathematics. His typology contains 4 different types of visualization based on a certain level of abstraction, more precisely the strength of the relation between the object and its visual mapping. We have to keep in mind that he does not use this typology in a clear way all the time: many visuals cannot be put into only one category or even not into any of them [16].

In Mathematics, two structures are isomorphic if there is an isomorphism from one to the other - which means that they are elementarily equivalent. Although the two structures look different, their elements are basically the same, while they have different names. So in *isomorphic visualization* a strong relationship can be found between the visual elements and the initial Math problem. That is why this category is the most frequently used in education. On the other hand it requires a specific sign system (which can depend on traditions, cultures, ages, ...) which is known by users. (Fig.1.)

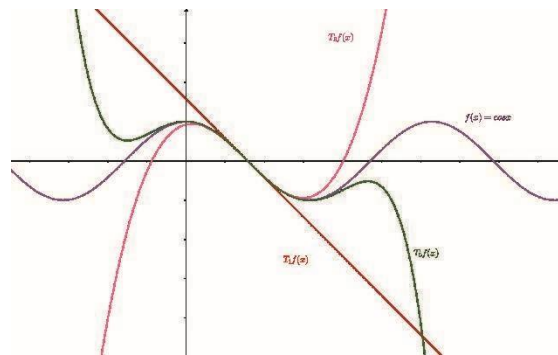


Fig. 1. Example for isomorphic visualization (Taylor polynomial)

When we look at two homeomorphic structures, the bases of the structures look similar, but the elements and operations are presented in a different form. In *homeomorphic visualization* the focus is on relationships: abstract objects can be depicted this way in order to have guesses or lead to proofs. Here, one cannot find a direct relation between the content and the visual presentation. That is why sometimes it is not easy to understand these structures, as it can often be subjective. (Fig.2.)

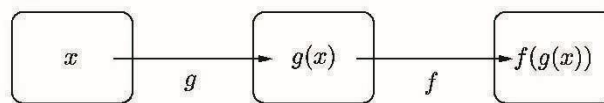


Fig. 2. Example for homeomorphic visualization (composite function)

The visualization which needs the most advanced understanding of abstraction is *analogical visualization*. In this case we swap the object mentally with another object that relates in an analogous way in order to work with it in an easier way. Nowadays this kind of visualization is rather rare, but Archimedes used it as an effective discovery method. (Fig.3.)

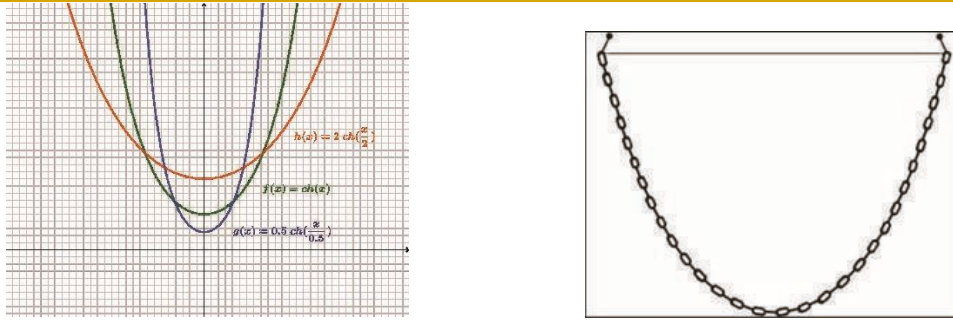


Fig. 3. Example for analogical visualization (catenary curve)

In the fourth category, diagrams are used to represent the relationship between mental objects. *Diagrammatic visualization* is frequently used in number theory and in probability. (Fig.4.)

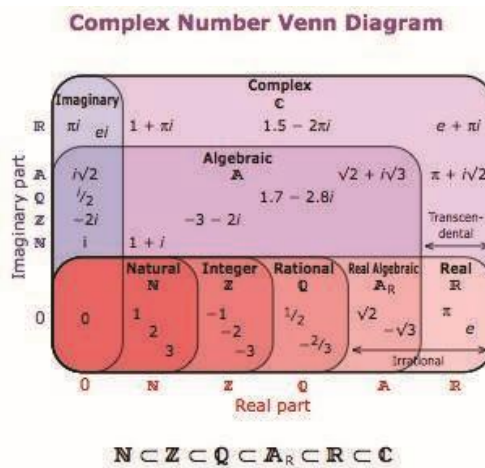


Fig. 4. Example for diagrammatic visualization (set of numbers)

Weinbrenner provided a complex model for textbook researches: he distinguished three categories: process-oriented (how a textbook is realized), product-oriented and impact-oriented (meaning what kind of effect a textbook has on culture, society, social norms). Here we focus on product-oriented textbook research in which textbooks appear not as a product on the textbook market but as a tool in education and visual communication. Comparative analysis can be performed in 4 dimensions: dimension of science theory, dimension of scientific disciplines, dimension of didactics and dimension of education. [17] Based on textbook researches, several didactical standards can be specified. Without completeness, textbooks should call for problem solving, call for action, fit to the psychical nature of students, be lifelike, and contain various kinds of methods. To continue the list we have to pay attention to the changed visual culture of students. Some recommendations for visual contents are listed by Dárdai [18]: the preferred ratio between visual and textual parts should be between 3050%. At first this ratio may appear high but when we consider our digital age, visuals dominate in every aspect of our life.

3.3 Mathematics curriculum from visual aspects

The European Society for Engineering Education (SEFI) published A Framework for Mathematics Curricula in Engineering Education which aims to provide guidelines for universities and educators in the topic of general mathematical

competencies for engineers, content-related competencies, knowledge and skills, teaching and learning environments, and assessment [19].

The document of SEFI highlights two main areas where visualization can appear effectively. Among 8 general mathematical competencies which are essential for engineers we can find the following competencies: representing mathematical entities, making use of aids and tools. Since mathematical entities can be represented in several ways (numerical, symbolic, visual, graphical, verbal, material objects), it is crucial to know their relationships and to be able to switch between them. Hence, a visual and graphical representation can depict phenomena from different aspects but it is worth to be aware of its limitations as well. On the other hand visual aids and tools can contribute to a deeper understanding of a complex concept or processes if the teacher utilizes their potentials.

Generally, visuals are created to explore, present relationships, data, results of some experiments, show statistics, but contrarily they can explain different processes and concepts as well. The former can appear in presentations, while the latter reinforces explanatory visualization. The method of presentation is more frequent in everyday life. Sometimes the difference between them is very slight. However the main aim of explanatory visualization is to educate, to enhance understanding. It focuses on theory and processes, the connection to other principles and interactivity is its own [20].

4 THE METHODOLOGY OF ANALYZING VISUAL ASPECTS IN MATHEMATICS TEXTBOOKS AND IN A CURRICULUM OF MATHEMATICS 1

Based on Guzman's typology and Dárdai's recommendations we investigated 10 textbooks [21-30] which were written for engineering students in higher education. The chosen books were written by Hungarian authors for Hungarian students. The subject of each book was Calculus which is a basic subject for each engineering student in the Bachelor level. Engineering Mathematics education has not changed in the past decades dramatically: the basics of the content in calculus remained the same, so it was suitable to analyse from a visual aspect. Older and newer textbooks were involved in our investigation in order to recognize some changes in visual representations. The newest book was written at Széchenyi István University after a curriculum development. With the exception of two textbooks - which are available online - they all have been printed versions. We used both qualitative and quantitative methods as well to recognize the nature of visualization.

Based on the framework of SEFI, on local needs (what does other teachers teach in other engineering subjects, time limit) and on learning outcomes theory we prepared new curricula in mathematics. We chose our new curriculum in Mathematics 1 for the investigation, which is taught in the first semester for all engineer students (vehicle-, logistic-, civil-, infrastructural-, computer science-, mechanical, electrical engineers, technical manager, architect) with the same learning outcomes.

5 FINDINGS ON VISUAL ASPECTS IN MATHEMATICS TEXTBOOKS

Firstly, we investigated the number of visual elements in the books which shows a very low number of explanatory visuals. Except two books, less than every second page contains a picture, the textual parts dominates on the pages. This finding does not satisfy the Dárdai's proposed minimum ratio which means there is a huge lack of

pictures. In some ways it is astonishing because calculus can be visualized very easily through functions in contrast with other more abstract topics. Textbooks can be divided into three category based on content. In the first category textbooks contain theoretical background of the topic with some examples which are solved [23, 25]. In the next category books contain only examples for individual practice [26, 29] and in the last category books contain theory, solved examples and examples for individual work as well [22, 24, 27, 28, 30]. This was important to mention in this context because there is a big difference between these categories in visualization. Those books which list only examples for individual work they almost do not contain visuals at all. In these books there were only few examples which required visuals to solve the problems, and there was any examples which presented a picture that students have to analyse in some way. In the other two categories books used more times pictures to illustrate the solution process or the solution. Only one book [30] uses pictures in that way students have to interpret, analyse or read some information from it. This leads to the conclusion that images are used to explain the meaning of a theory, or to visualize the solution of an example but students are not taught in the investigated textbooks how to interpret an image which would presume a deeper understanding from them.

Secondly, images investigated by Guzman's typology do not show a wide range of variety in visualization. We could find schemas how they visualize a content, theorem. As Guzman mentioned, the most frequently used type of visualization is isomorphic visualization in which there is a strong relation between the math content and image. That is the reason it can be understood easily in most cases. Then homeomorphic visualization was the second most common, while we could find two examples for diagrammatic visualization and only one for analogical visualization. In addition to mathematical tasks, each book has engineering-related tasks as well. These technical tasks often use figures to illustrate physical changes. E.g. work, power, torque, magnetic field, electrical consumer, movement, spreading of sound, light can be presented mostly in homeomorphic way which need more abstract thinking skill. *Table 1.* lists which content was visualized with which type of visualization. *Table 1.* Content and the connected type of visualization

Type of visualization	Content
Isomorphic visualization	Graph of a function, function transformations, inverse function, monotonicity, local extrema, convexity of a function, inflection point, limit of a sequence, meaning of index n_0 , limit of a function, prove a notable function limit, tangent line, secant line, meaning of derivative, definition of continuous function, intermediate value theorem, approximation with Taylor polynomial, Rolle's theorem, geometrical examples to find local extrema, definite integral as the limit of a sum, area under a curve, area between two curves, volume of revolution, technical problems
Homeomorphic visualization	Domain, range of a function, how to create a composite function, an inverse function, approximation with iteration, technical problems
Analogical visualization	Work as area under a curve
Diagrammatic visualization	Venn diagram of composite function, diagram of continuity, differentiability, integrability and boundedness

We conclude these findings jointly with the findings of the Curriculum analyses at the end of this paper.

6 FINDINGS OF A CURRICULUM DEVELOPMENT FROM VISUAL ASPECTS

In the subject Mathematics 1, main topics are calculus and coordinate geometry. Some topics are more abstract than others but these topics can be easily visualized. In *Table 1*, we highlighted that part of the curriculum which details the acquired knowledge and skills during the subject. We made some visual suggestions connected to the listed knowledge and skill elements and specified those visual representations which can be supported by presentation or explanatory visualization. If we base the mathematical knowledge from different aspects at the beginning well, it is easier to build on it later. Visual aids can contribute to deeper comprehension that is why we must not forget the potential of visualization.

Table 1. Learning outcomes with the connected visual representations

Knowledge	Skills	Presentation (P) or explanatory visualization (EV)
As a result of learning this subject the student should ...	As a result of learning this subject the student should be able to ...	
know the concept of spatial vector, operations with vectors (addition, subtraction, multiplication with scalar), product of vectors (dot product, cross product, triple scalar product) and their properties	solve tasks related to analytic geometry	discover the relationship between spatial element with the help of some sketches (P, EV)
know the equation of line in space and equation of plane		
know the interrelationship of spatial elements, the methods of calculating the intersections of spatial elements		
know the calculation of distance between spatial elements		
know the calculation of angle enclosed by spatial elements		
know the theory of complex numbers, the rectangular and polar form of them	convert between different forms of complex numbers	complex numbers among set of numbers (P) sketch complex numbers in complex plane (P)
know the operations with complex numbers and their properties	calculate the roots of real and complex polynomial solves different types of equations among complex numbers	sketch the meaning of different operations with complex numbers in the complex plane (EV)
know the real univariate functions and their properties		sketch the graph of functions (P)

know the concept of composite functions and inverse of real univariate functions, inverse of elementary functions	determine the composition and inverse of real univariate functions	sketch the inverse function from the original function (reflect over the $y=x$ axis) (P, EV)
know the graph, domain and range of real univariate functions	determine the domain of real univariate functions, determines the range of a function from its graph	determine the domain and range of functions from their graphs (P)
know linear transformations and their effect on the graph of real univariate functions	sketch functions by linear transformations	understand how linear transformation alter the graph of function (P)
know the concept and characteristics of limit of sequences, the limits of the notable sequences, the critical limits, and the concept of index n_0	calculate the limit of sequences, examines convergence, divergence	explain the visual representation of limit (EV) and the concept of index n_0 (EV)
know the concept, properties and meaning of limit of real univariate function	calculate analytically or read from the graph the limit of real univariate functions	present the illustrative meaning of limit of functions, meaning of convergence, divergence (P, EV) prove notable special limits through geometry (EV)
know the concept, properties and meaning of continuity of real univariate function	analyse continuity of functions	present the illustrative meaning of continuity of functions (P)
know the basic concepts of differential calculus and differentiation rules , derivatives of elementary functions	calculate the derivative of a function, the higher derivatives of a differentiable function	explain the derivative of a function as the instantaneous rate of change (EV)
know the geometric meaning of derivative of univariate real functions, the equation of tangent line	write the tangent line to a given function	explain the connection between secant, tangent line and derivatives (EV) draw the tangent line to a function (P)
know Taylor Polynomial and Maclaurin Polynomial	approximate differentiable functions with the help of Taylor Polynomial and Maclaurin Polynomial	present the graphical approximation with Taylor- and Maclaurin Polynomial (P)
know the L'Hospital rule	apply the L'Hospital rule when calculating limit values	
know the concept of monotony, local and global extremes and relationship with the first-order derivative	apply differential calculus to determine the shape of a function and local extreme values	draw the function based on the calculated properties (P)
know the concept of convexity, inflection point and its relation to the second derivative	apply differential calculus to determine convexity and inflection points	
know the following concepts: parity, zero, intercepts, monotony, convexity, asymptote	examine functions from different aspects in order to graph the function finally	

know Riemann integral, the meaning of definite and indefinite integral, fundamental theorem of calculus	calculate definite and indefinite integral of integrable functions	explain the idea of a definite integral as the limit of a sum (EV)
know the most important methods in integration		
know the most important geometric applications of integration (area, volume of revolution)	use integration in vocational subjects to determine areas, volumes and center of gravity	draw the calculated area (P) explain the calculation of volume with the method of cylindrical shells (EV)

7 CONCLUSIONS AND FUTURE PERSPECTIVES

In our visualized, digital age the Mathematics teaching needs to be innovated. With analysing the relevant and fresh interdisciplinary literature (including adult learning theories, findings in brain research, visualization and new guidelines for curriculum development of SEFI) we conclude that according to our described conceptual framework, a research-based implementation of as many types of visualization as possible in teaching Mathematics in engineering education would reduce the proportion of unmotivated and unsuccessful students in Maths. Visual aids can offer extra help in understanding especially for visual type of learners. The current generation is visually literate so visualization can contribute to a deeper and better comprehension in mathematics for them. Moreover, our approach can provide extra motivation for engineers because later in real life visual communication skills can support communication with specialists from other fields as well.

We can create a new term of “visualization-diversity” expressing focuses on visualization-oriented, revised and innovated textbooks and curricula (including visual, auditive and animated digital learning materials with all 4 types of visualization of Guzman typology), utilising videos on tablets, smart-phones and other devices which are available for the engineering students. Nowadays mathematics software programs make visualization easier but so far we have not used of their full potential in education. A further opportunity for smart learning could be along with self-directed learning the collaborative learning of virtual communities if students (and teachers) are involved in creating and discussing new learning materials to support the learning of all participants.

The above-mentioned perspectives and the detailed findings of this work can contribute to developing an up-to-date conceptual framework for improving the quality of teaching Mathematics in engineering education at a Hungarian university, and it might be useful for other universities as well.

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