

English Language Support in Statistical Education: Investigating the Needs of Non-Native English Speakers

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Abstract

Despite the rapidly growing population of English Language Learners in schools, very little research has focused on understanding the challenges of English Language Learners in statistics education. This paper reviews research by statistics and mathematics educators to highlight some of the broad challenges faced by English Language Learners in statistics learning and teaching. The linguistic challenges include the vocabulary in academic statistics and linguistic features that may make statistical texts hard to understand and communicate. Next, the review outlines pedagogical strategies to help learners in statistics classrooms. The final section considers some issues arising out of the review and offers suggestions for practice and research.

Keywords: English Language Learners, Statistical Language, School Students, Multilingual Settings, Language Barriers, Strategies, Implications

Introduction

“Imagine a teacher running her hands across the table as she tells her students “A variable is any characteristic, number, or quantity that can be measured or counted.” The students listen quietly, but one of them is thinking, “I think variable was a letter which can stand for many different numbers such as $y = x$ ”

A common view about mathematical language is that it is culture free and therefore a universal language (Brown, Cady, & Taylor, 2009; Lesser & Winsor, 2009; Meaney, 2006; Planas & Setati-Phakeng, 2014; Sharma, 2014; Waller & Flood, 2016). However, research indicates that the language of mathematics/statistics is intrinsically cultural which can be confusing and very challenging for students (Bay-Williams & Herrera, 2007; Groth, Butler, & Nelson, 2016; Kaplan, Fisher, & Rogness, 2009; Lavy & Mashiach-Eizenberg, 2009; Lesser & Winsor, 2009; Moschkovich, 2018). Students need to recognize that terms such as random, inference, and normal have specific meanings in statistics that may differ from everyday usage. It is not only the meaning of words used within a statistical context that can be confusing. In the quotation above, the mathematical usage of the term; "variable" can suggest algebraic equations where letters stand for different numbers, these letters are considered variables, there is no variation within the statistical sense. As these brief examples show grappling with how language is used in statistics can be difficult for a number of students (Groth, Butler, & Nelson, 2016; Kaplan, Rogness, & Fisher, 2014; Kazima, 2006; Lesser & Winsor, 2009; Sharma, 2014). However, to understand mathematics and statistics in an English medium classroom, English Language Learners may have to undergo more processing than native English speakers (Barwell 2005; Kazima, 2006; Latu, 2005; Lesser, Wagler, & Salazar, 2016; Makgato, 2014; Meaney, 2006; Moschkovich, 2005).

While the role of language in the teaching and learning of mathematics is now well established in the literature (Bay-Williams & Herrera, 2007; Clarkson, 2007; Hoffert, 2009; Moschkovich, 2007; Planas & Civil, 2013), there have been a few research studies about language issues in learning statistics (Kaplan, Fisher, & Rogness, 2009; Lavy & Mashiach-Eizenberg, 2009; Parke, 2008; Rangelcroft, 2002) but these did not involve English Language Learners.

It is important to gain insights into how English Language Learners learn statistics (Kazima, 2006; Lesser et al., 2016; Lesser & Winsor, 2009, Sharma, 2014). Given the dearth of research on English Language Learners learning statistics, this review provides much-needed contributions and stimulus for more debate and research in this area. The paper draws on mathematics and statistics education research to discuss the challenges faced by English Language Learners. The next section discusses some strategies teachers can use to support learners. The final section considers the issues arising out of the review and offers suggestions for practice and research.

Literature Review

Problems Faced by English Language Learners

It is well known that, communication in English can be a barrier for many English Language Learners because English serves both as a content subject and also as the means of pedagogic interactions in classrooms. This section is organised around six sections. It must be noted that this way of organising the review is a simplification as there are overlaps across the sections.

Word meanings: Ordinary English verses Statistical English

To be able to communicate statistically both verbally and in writing, students must understand the highly technical language used specifically in statistics (Gal, 2004, Kaplan, Rogness, & Fisher, 2014; Parke, 2008; Watson, 2006). This language may be used in everyday English, and therefore is likely to be familiar or understood by English Language Learners (Groth, Butler, & Nelson, 2016; Lavy & Mashiach-Eizenberg, 2009; Lesser & Winsor, 2009). One example of confusion between the everyday and statistical English (Lesser & Winsor, 2009) involves the term independent. Since the everyday meaning of independent can be associated with separateness (e.g., independent nations), the authors conjecture that this leads students (incorrectly) to equate independence with disjoint (i.e., mutually exclusive) Another technology related example provided by Lesser and Winsor is the term mode. The mode button on a calculator has nothing to do with the most frequent observation in statistical context. Pfannkuch (2011) gives the example of learners referring to “the average student,” possibly due to its usage in the everyday media.

While some terms have different meanings in everyday usage and statistics, some are also used in mathematics in more than one way (Kaplan, Fisher, & Rogness, 2009; Lesser & Winsor, 2009). Lesser and Winsor (2009, p. 12) provide an example of this confusion:

I: What is the range of this data set?

(1, 2, 3, 4, 6, 6, 13)

S: Seven

I: Okay how did you get that?

S: Just the number of the elements.

I: How many values in the above dataset are at most six?

S: One. The only number that is greater than six is 13.

In the example above the confusion can be read in two ways. The everyday usage of the term 'range' can suggest 'ranging through' the 'full range of' elements of the data set, thereby evoking the concept of sample size. However, it is possible that the student's use of all the numbers in the data set in this case could equally reflect their interaction with a mathematics register in which the word 'range' is generally used to indicate a set of numbers rather than a single number. Even symbols (e.g., N , p , α) can be used in more than one way within the area of statistics.

Groth, Butler, and Nelson (2016) reported that 11-12-year students can struggle to understand and use terms that describe probabilities. Such struggles lead to difficulties comprehending classroom conversations. The authors describe some specific misunderstandings a group of students (ages 11–12) held in regard to vocabulary such as certain, likely, and unlikely. Kazima (2006) study explored the meanings that, prior to instruction, students assign to some words such as impossible, never, unlikely, that are commonly used in teaching probability. The sample for the study consisted of 154 students in their first year of secondary school education and whose first language was Chichewa. The research instrument required students to offer meanings for words where no context was given. The findings demonstrate that many of the students' preconceived meanings for probability vocabulary were distant from established conventional probability meanings. In addition, there was a wide range of meanings associated with each of the words.

The above situations may present some unique challenges for students as they must simultaneously learn ordinary English and statistical English and be able to differentiate between the types of English (Kaplan, Fisher, & Rogness, 2009; Kazima, 2007; Lesser & Winsor, 2009; Nacarato & Grando, 2014; Watson, 2006). To understand statistics in an English medium classroom, English Language Learners may undergo more processing than native English speakers (Clarkson, 2007; Latu, 2005; Meaney, 2006; Schleppegrell, 2007). These students can miss out on learning because they may be spending too much time trying to understand the problem or the question.

Statistical Register

Within any particular language, there are many distinct registers, including everyday conversation, mathematics, statistics, and so on (Gibbons, 2008; Marin, 2018. Schleppegrell, 2007). Halliday (1974) used the term *register* to refer to the specialised method of communication used in a particular social practice.

It follows that in statistics classrooms, multiple registers are used. To succeed in a statistics classroom, students need to not only be familiar with and competent in their ordinary English register, so they can communicate with their classmates, but must also have fluency in what can be termed multiple mathematical registers (Kazima, 2006). The mastery of the statistical and mathematical registers, and the strong ability to switch between them, requires strong linguistic and metalinguistic skills (Lavy & Mashiach-Eizenberg, 2009; Kaplan, Rogness, & Fisher, 2014; Lesser & Winsor, 2009; Schleppegrell, 2007).

Schleppegrell (2007) claims that technicality can also be conveyed in grammatical choices in mathematical texts including long, dense noun phrases and relational and attributive phrases, such as "the volume of a rectangular prism with sides 8, 10, and 12 cm" (p. 143). A statistics example could be "the upper quartile of a normal distribution with mean 75 and standard deviation 10." Kaplan, Fisher, and Rogness (2009) and Kaplan, Rogness, and Fisher (2014) considered the role of lexical ambiguities in the statistics classroom. Kaplan et al. (2009) define words that lack a core meaning as lexically ambiguous and suggest that ambiguous words such as "spread" should be avoided.

Lesser, Wagler, and Salazr (2016, p. 149) state that, probability learning is greatly affected by matters of registers, often in ways that are more difficult for English Language Learners. They state that many probability statements use words such as "at least," "at most," "fewer than." They add that students accustomed to a reflexive "keyword"

association (e.g., treating "at least" as "less than") may make errors interpreting a question such as "What is the probability that Bob will have at least 8 successes in 10 attempts?"

According to Schleppegrell (2007), the mathematics register is challenging for all students, and especially for students with few opportunities to use academic registers outside school, including "speakers of nonstandard varieties of English and students for whom English is a second (or other) language" (p. 153). For an English Language Learner, statistical registers can pose a significant challenge, as a new form of language must be learned and mastered (Mandy & Grbati, 2014; Marin, 2018; Moschkovich, 2005; NCTM, 2008). Not only must English Language Learners try to learn in English whilst concurrently learning to speak English, but they must also be working within the English statistical registers without yet having mastery of ordinary English.

Even if an English Language Learner is competent in using the ordinary English register, the use of the statistical register can provide extra difficulties. Just because someone learned statistics in one language and is conversationally fluent in another language, it does not mean they can communicate about statistics in that latter language (Moschkovich, 2002).

Reading and Writing

Statistics educators (Gal, 2004, Sharma, 2018; Watson, 2006) state that literacy skills are critical for statistical literacy because virtually all statistical messages are conveyed through written text or oral text. Gal (p. 4) writes that "some messages also require that individuals navigate through and comprehend displays that employ a combination of non-prose text and symbolic numerical information, such as tables, graphs or charts." Gal adds that many statistical terms such as sample and correlation often cannot be represented in numbers when discussed in an everyday context and are therefore communicated daily through words.

Reading statistical texts provides the learner with an extra challenge over reading English (Benjamin, 2011). The learner must simultaneously comprehend and process in both the language of English and the language of mathematics (Hoffert, 2009; Kester-Phillips, Bardsley, Bach, & Gibb-Brown, 2009). Additionally, student when reading textbooks, they rely heavily on their knowledge of vocabulary and linguistic conventions used to make texts explicit and self-contained (Schleppegrell, 2004). Language in traditional classrooms is more context-reduced, so students have few contextual cues to help negotiate meaning.

According to (O'Halloran, 2005), the mode dimension can impact language choices as well. In oral interactions, there are frequent opportunities for immediate feedback, and dialogic, co-construction of ideas, whereas, in writing, a writer expands and develops ideas individually. Statistical writing, moreover, characteristically draws on multiple forms of representation, including symbolic and visual representations (Gal, 2002). All teachers in Sharma (2018) study mentioned students' difficulties with reading, speaking, and writing in English. This affected their ability to engage in written work. For example, according to Teacher A (TA), this could be due to students not using English language at home.

Students don't want to write down...they are struggling to speak, read, and write. They remain silent even though they miss key phrases. They speak their own language at home even when doing homework, they may not speak English. (TA)

In contrast, Teacher C's (TC) reflection suggests that sometimes mathematics teachers might not have the skills to teach the written component of statistics.

Mathematics teachers find it hard when you got to do scaffolding. We are not natural teachers of writing. It is okay in mathematics, and then as maths teacher we are not good at a particular way of writing and helping with statistics requires a different way of writing. (TC).

Redundancy is another characteristic of ordinary English that has a significant influence on how students (mis-) read statistical English. Ordinary English has a high degree of redundancy consequently students learn to skim read, sampling keywords to get the key point, for example, when reading a novel. In comparison, mathematical

English is concise, each word has purpose with little redundancy, and a large amount of information is contained in each sentence – sentences are dense (Padula et al., 2001). Students who transfer their reading skills from ordinary English to mathematical English texts may be disadvantaged by a tendency to overlook key information. Cultures with less redundant natural languages are more likely to pay attention to every word and therefore understand better some forms of mathematical English despite this being their second language (Latu, 2005).

Sharma (2014) reported that every day reading strategies of skimming and using the context or knowledge of the world to support comprehension are insufficient for reading statistical English. For example, the meaning intended by Sharma on the interview tasks was not that constructed by the students. As a result, students constructed responses based on these unintended interpretations. Rather than attending to proportionality information given on a marble task, (10 white and 20 black marbles versus 20 white and 60 black marbles) some students based their reasoning on their everyday language skills. Ordinary expectations of the way words are put together in English led students to assume that the phrase *at the same time* in the question will mean some action.

Role of context

Context refers to the setting in which information is communicated and may include content, people, or environment. The vital role that context plays in statistics education has been discussed by statistics educators (Cobb & Moore, 1997; Gal, 2004; Lesser & Winsor, 2009; Neill, 2012; Scheaffer, 2008). Neill (2012) argues that in statistics the context is at the heart of any investigation. All aspects of a statistical investigation must directly relate to the context in which the investigation is situated. For example, take the context of shoes. In the question, "I have 4 shoes and get 3 more, How many shoes do I have now?" the context is incidental to understand the mathematical problem and shoes could equally well be marbles or apples. However, in the question, "What is the most common type of shoe in our class," the context of shoes plays vital role in all aspects of the investigation.

According to Lesser and Winsor (2009), the most important clue to deduce the meaning of a word or sentence is generally its context, and when students find context meaningful, it increases their motivation to learn and communicate.

Goldenberg (2008), claims that statistics is a more natural vehicle for context-embedded instruction than mathematics, hence context embedded instruction can have benefits for teaching ELLs statistics.

Lesser and Winsor (2009) reported a student's confusion from a context-rich exercise about correlation because the term "ski resort" was unfamiliar in her high-poverty urban city in a desert region. Sharma (2018) reported that students, when carrying out statistical investigations focussed on real-life contexts, can often get side tracked by irrelevant details while ignoring relevant information. For example, some students in a teacher's class in Sharma study interpreted kiwi birds as kiwi people.

The findings indicate that there may be a tension in that an ELL may struggle to learn if there is no context given and yet there is also an obstacle if a teacher offers a context the ELL finds unfamiliar. One explanation for this conflict could be situatedness is the idea that people make sense and behave differently when situated in different practices. Using a word in its everyday sense may thus be seen as the result of failure to recognise the situation as mathematical rather than failure to distinguish the correct mathematical sense of the word (Moschkovich, 2002).

Code Switching

English Language Learners are known to employ code-switching to clarify their understanding and to express their arguments and ideas generally (Clarkson, 2007; Moschkovich, 2005). Code-switching involves the movement between languages in a single speech act. It can involve switching a word, a phrase, a sentence, or several sentences (Adler, 1998). Bose and Choudhury (2010) state that in addition to a switch between two languages, for example, English and Hindi, the teacher also switches from a formal version of Hindi to a very colloquial form of the same language. In this case, the code switch takes place as a language-swap from English to Hindi, as well as from the

formal form to an informal form within one language. Magid and Mugaddam (2013) assert that code switching can take place in a conversation when one speaker uses one language, and the other speaker answers in a different language.

Code-switching has been shown to promote English Language Learners student-student and student-teacher interactions (Kasmer, 2013; Setati et al., 2002). Sometimes students switch their languages because they find problem difficult to solve in English. Clarkson (2007) explains how English Language Learners may comprehend target language texts using their first learnt language (L1). He claims that the first language scaffolds semantic processing, while if a learner were to process the input exclusively in second language/formal language of instruction, then s/he might run into trouble handling syntactically complex sentences. Makgato (2014) found that code switching to home language is a common practice in order to sustain continuous communication between teachers and learners.

Research studies provide examples of the tension between use of code switching and school languages. Clarkson (2007) explains how translation is not always beneficial or reliable as it might not reflect the exact meaning. Thus, switching between languages can add an extra layer of challenge to language learners, as they may find themselves working between a multitude of registers in both the medium of instruction and their home language (Mady & Garbarti, 2014; Schleppegrell, 2011). Moreover, students' use and perceptions of the value of a particular language in different settings varies. For example, Planas and Setati-Phakeng (2014) reported that while students used their home language in small group settings, they did not do this during whole-class discussions.

Although students could explain their thinking in their mother tongue, in Sharma (2014) study, none of the students used this opportunity. At home, students mostly use their mother tongue in their speech which is considered pre-literate, and hence, students did not consider vernacular an appropriate language to use in this context. When students are learning statistics, teachers need to understand that there may be no suitable vocabulary available in their home language. For example, there is no equivalent term for "probable" in the Tongan language (Latu, 2005).

The difficulties noted in this section may present some unique challenges for all student. However, to understand statistics in an English medium classroom, English Language Learners may have to undergo more processing than native English speakers (Barwell, 2012; Kazima, 2006; Latu, 2005; Meaney, 2006). The linguistic complexity English Language Learners face demonstrates the need for strategies for supporting English Language Learners.

Strategies in Multilingual Classrooms

This section describes some language strategies that teachers can use to address some of the linguistic challenges faced when the language medium of instruction is different to the home language(s) of students in an educational setting.

Integrating reading and writing

Supporting English Language Learners in reading and writing is vital in multilingual settings. It is important to have daily routines of writing, reading and speaking about statistics content (Hoffert, 2009; Sharma, 2018; Sharma, Doyle, Shandil, & Talakia'atu, 2011; Winsor, 2007). Statistics writing may be informal (e.g., journals, exit slips) or formal (e.g., writing reports) (Hebert, & Powell, 2016; Winsor, 2007). Although a teacher expressed concerns about integrating writing in statistics in Sharma (2018), it was clear that the teachers were supporting their students in their writing by integrating language strategies and statistics content in their lessons, an important component in statistics lessons and, in particular, statistical projects. All reported that they had to write and draw a lot on the whiteboard. They used class notes so the students could follow what was being discussed. Writing the key terms helped students see them and connect them to the spoken word, as indicated below:

They need practice in both reading and writing. I give them time to read what is written on board or in their class notes before I start talking (TC).

All teachers used techniques to make sure all students understood the instructions. Teacher B (TB) gave explanations and instructions in clear and simple language and then asked a student to repeat the explanation.

I try to give instructions step-by-step before asking students to do independent, pair, or group work. Then I ask one of the boys to repeat the instructions aloud for the rest of the class to make sure all have understood what is required (TB).

In her whole class sessions, TA slowed down her speaking pace a bit while TC reported modifying the linguistic complexity of his speech by using shorter sentences and re-phrasing questions. As well as modifying speech, TA also wrote notes and questions on a mini white board she used during her small group interactions.

Both TA and TC also provided a writing frame and cloze activities to help students analyse data and draw conclusions. For example, the following writing frame was provided for writing conclusions about kiwi population data.

The evidence from mysuggests that of kiwis is betweenand
Approximately of kiwi isbetween..... and

Students had to complete the statements from their sample results and then wrote a statement about what they thought might be happening. Some statements were collected on the board so that the class could look at the overall results.

Student journals offer another way of listening to student communicate mathematics and often be used to capture ideas recently addressed in class (Benjamin, 2011; Hoffert, 2009; Winsor, 2007). Winsor used writing in journals strategy to help English Language Learners learn mathematics and the language of mathematics at the same time. Students were allowed to write in the language they felt comfortable with, but they were required to write the mathematical terms in English. Using the mathematical terms in English helped students associate the English term with the mathematical concept already in their minds. At the end of each week, students evaluated their peers work using a simple three-point rubric Winsor claims that the students in the study benefited in a number of ways. Some writing prompts that could be used in a statistics lesson are included in the box below:

<p>What is the difference between the range of a data set and the range of a function? Compare with what random means in everyday English with what it means in statistics. Complete the following analogy and explain your thinking. Mean is to descriptive statistics as range is to.....I thought a sample wasNow I know that a sample is....Students can peer edit and evaluate one another's writing.</p>
--

Groth, Butler, and Nelson (2016) asked students to write a letter to a student who had been absent, explaining the meanings of probability terms from the lesson. All the students in their sample included correct explanations for the benchmark terms, certain, impossible, and evenly likely. From the students' writing, the authors found that students were assigning incorrect numerical probabilities for in-between terms such as unlikely and almost impossible. The presence of this pattern in students writing made the authors aware of the need to emphasize the distinction between the benchmark and in-between terms in future lessons.

Using non-linguistic cues

Providing non-linguistic cues such as visual diagrams, demonstrations, physical items, and gestures can make more complex language accessible for all learners as these are less language dependent modes. Murrey (2008) suggests that having such multiple entry points and scaffolds are helpful not only in the direct way of communication but also in the indirect way of helping create a low-anxiety environment.

Visual diagrams such as graphic organisers can be especially beneficial when the graphic organisers are allowed to be filled in in both English and the student's home language (Kaplan, Rogness, & Fisher, 2014; Ngyuen & Cortes, 2013; Winsor, 2007). Graphic organisers can enable English language learners to see the relationships between key mathematical concepts and vocabulary. In the case of words with multiple meanings it is important to emphasize the similarities and differences, so the student can assimilate them. This helps them become flexible, adaptable thinkers, (Benjamin, 2011). Compare and contrast activities like the graphic organiser below can be used for this purpose.

Statistics meaning	Other meaning(s)
Picture/Image	Picture Image

Offering physical items and demonstration alongside verbal explanation, for example. holding up three fingers when discussing three points, is an effective technique to scaffold English language learners (Brown, Cady, & Taylor, 2009). Year-12 students in Sharma (2018) study had difficulty posing good statistical question from given data. Teacher modelling helped students realise statistical questions can be classified into three categories. Hoffert (2009) used a similar strategy. She gave each student a question on card with magnets on the back and had the class sort these questions into three categories without any specific explanations. Students who did not speak the same language tried to communicate with one another while sorting. Hoffert (2009) claims that since they developed their understanding in a group activity, they remembered the three types of questions.

Furthermore, the use of physical or pictures can support students in their comprehension of statistical terms. (Brown, Cady, & Taylor, 2009; Nguyen & Cortes, 2013; Weist, 2008). One fundamental word in statistics that has lexical ambiguity for students is "random."(Kaplan, Rogness, & Fisher, 2014). . Following the activation activity, the instructor in this study showed her students two pictures. The first was of three people dressed in rainbow-striped zebra costumes on a street in Shanghai to represent the colloquial definition of *random*: something that is weird, haphazard, or out of the ordinary. The other was an upside-down hat to represent the statistical definition of *random*: where choices or outcomes are based on probability. This introduction provided the instructor with the zebra-versus-hat mnemonic image for *random* that she used during the rest of the semester to contrast the statistical and colloquial meanings of *random*.

Gestures are important for leaning for a variety of reasons (Benjamin, 2011; Cook & Goldin-Meadow, 2006; Moschkovic, 2002). Gestures can clarify language and improve the likelihood that the language will be remembered. Gestures can consolidate related information in learning, allowing the learner to distinguish between important and secondary information. When we support language with gestures, we engage in the spatial, visual, and kinesthetic neural pathways. As another possibility, gesture uses the body to do its representational work, and these embodied representations might promote learning Cook and Goldin-Meadow (2009, p. 229) state:

There is increasing evidence that embodied forms of representation are involved in cognitive processes, including working memory, mental imagery, and linguistic processing.... Gesture, as an embodied representational format, could preferentially engage any one or all of these four systems in contributing to learning.

The teachers in Sharma (2018) used strategies that supported students visually and helpful in scaffolding students who may not have the language skills to match their statistical ability. A teacher used his thumb and index finger to show the different ranges as he moved along samples of 15 and then 30. The students could notice that with the samples of 15, the teacher's fingers moved in and out a lot whereas, with the samples of 30, there was little movement. Hence the range of the values in the smaller sample vary a lot more than in, the larger sample.

Focus on statistical reasoning

To best support the statistical development of English Language learners, including their development of statistical language, there should be a primary focus in statistical reasoning, as opposed to language accuracy (Barwell, 2005; Moschkovich, 2013). The use of journals can enable this. When an English language learner is able to explore and express their mathematical ideas in a journal, it gives learners who otherwise might be too shy or unsure to express themselves orally an opportunity to for expression, where the focus is on the mathematics, not on their language skills.

One example consistent with this perspective could be an ELL who, when asked to describe when the mean could exceed the median, is not able to state a phrase such as “a unimodal, right-skewed distribution,” yet communicates the essential idea with informal language or by drawing or tracing the shape (Moschkovich, 2015). This mathematical reasoning should not be restricted to the specific reasoning taught in class. Instead, all students, including English language learners, should be encouraged to solve problems using the techniques and strategies they know and are most familiar with (Brown, Cady, & Taylor, 2009). Through encouraging and accepting diversity of strategies and techniques, all students are exposed to a variety of strategies, therefore enriching their own mathematical understanding of mathematical processes. Furthermore, through embracing diverse strategies, including those different to the ones taught in class, English language learners are not seen as different for solving problems in a different manner to their peers. Instead they are accepted alongside their peers in sharing valid mathematical strategies.

Using home language

A number of international agencies recognise the contribution that multilingual education can make to engaging diverse learners. In addition to supporting academic achievement, students using multiple languages can also assist in the development of positive identities associated with home culture (European Commission, 2015; UNICEF, 2016), which is a basic human right. A number of researchers in mathematics education (Kasmer, 2013; Planas & Civil, 2013; Setati et al., 2002; Winsor, 2007). have identified that students’ home language(s) can serve as a resource for thinking and communication as students simultaneously learn and develop proficiency in the language of instruction and learn mathematics. Lesser, Wagler, and Salazar (2016, p. 146) state:

"Full participation of ELLs into the learning community is essential, not only with regard to issues of equity, but also to recognize the assets ELLs bring to a classroom environment."

While research shows that many teachers believe using home language is detrimental to learning (Mady & Garbarti, 2014; Planas & Setati-Phakeng, 2014; Winsor, 2007), this was not the case for the three participants in Sharma (2018) study. Students were supported by teachers and peers to use their home language, English, and mathematical/statistical English to discuss and develop their understanding about statistics. The home language might have served as an important scaffold for the accomplishment of the data-driven statistical task in English. However, these findings show that students need time to feel confident enough about using their home language in a group setting. The sense of stigma, as well as lack of mathematical reasoning in English, has to be overcome.

It is also worthwhile to identify ways in which features of an ELL’s language can help reinforce concepts of statistics. For example, Bannon (2007) notes that in Malay, the expression for the mean is *sama rata*, which translates roughly as “same level.” Thus, the language invokes the “leveling” conceptual interpretation of the mean. This interpretation is also reflected in the Arabic origin of the word “average” (Konold & Pollatsek, 2002). Additionally, because ELLs are used to having to go back and forth between English and Spanish, they may be more primed than monolinguals to navigate among various academic registers.

Collaborative learning

In order to enhance the communication skills of students, it is recommended that teachers increase dialogue in English by means of small-group discussion, exploratory talk, and argumentation. Collaborative learning is a powerful tool for all students, but especially English language learners (Takeuchi, 2016). When English language learners are able to work alongside a partner, they are given the opportunity for interaction and support, enhancing

their learning (Brown, Cady, & Taylor, 2009). Collaboration affords English language learners the chance to ask questions and make mistakes in a safe setting, where they can receive direct and immediate feedback (Weist, 2008). Furthermore, when students are engaged in authentic conversation and interaction, it best fosters their language development. This is especially true when English language learners are partnered with a peer who has a higher degree of English language proficiency (Takeuchi, 2016; Weist, 2008).

Collaborative work allowed the students to collaborate in their learning and ties in with the work of Brown, Cady, and Taylor (2009) and Winsor (2007) who explain that when language learners are able to work alongside a partner, they are given the opportunity for interaction and support, enhancing their learning. Goldenberg (2008) reported that collaboration could afford language learners the chance to ask questions and make mistakes in a safe setting, where they can receive direct and immediate feedback. This is especially true when language learners are partnered with a peer who has a higher degree of language proficiency in the language, which is the medium of instruction.

Hands-on Activities

Wells and Narkon (2011) and Rubenstein and Thompson (2000) write that hands-on activities such as games, sorting and matching activities and puzzles not only pique interest but also reinforce statistical topics. They add that, for a richer learning experience, students can be invited to create their own set of equivalent cards for new terms and symbols they have learnt. Card games can be a fun way of helping students to link verbal, symbolic, graphic, or story representations of the same values and concepts.

Two teachers in Sharma (2018) study used games and matching activities to help Pasifika students develop their statistical vocabulary. One teacher used a fun game called *Forbidden Words* to start or end a lesson. The idea of the game was for one player to try and describe a statistical term or phrase without using certain **forbidden words**. The other players had to try and guess the word. For example, **Fila** picks out word card - 'mean.' He then has to describe the word without using the words average, total, and divide. Others try to guess the word. An adapted version of the game could go like this:

Fila: It is a summary statistics

Tevita: range

Fila: It is a measure of centre

Tevita: median

Fila: ... and it uses all data

Tevita: mean

To make the game easier teachers could allow students to use one of the forbidden words or have a scoring system based on the number of forbidden words used. Additionally, students could be given a list of key statistical words and asked to make their own forbidden cards.

Another teacher used a matching activity to help student's statistical vocabulary. Students were provided with sets of word and description cards and asked to work in pairs and sort statistical terms with their descriptions. For example, the term "inference" was matched with its description "the process of drawing conclusions about population parameters based on a sample from the population." Once an agreement was reached, they could discuss their answers with another pair of students. Another matching activity involved students matching words, graphs with their descriptions. The activities helped students make connections between different representations.

Code switching

English Language Learners may code switch for various reasons, including to seek clarification and to provide an explanation. Code switching promotes both student-student and student-teacher interactions in classrooms involving English Language Learners (Kasmer, 2013). Code switching can be used strategically and

advantageously by teachers as a way of utilising a student's home language as a resource (Kasmer, 2013; Winsor, 2007).

Plana and Civil (2013) reported that teachers and students switched between languages in mathematics lessons because learners communicated their mathematical thinking more easily in their home language and that this switching improved mathematical dialogue in the classroom. Similarly, Clarkson (2008) reported that language switching helped Vietnamese students gain more confidence and hence enhance their mathematical learning. Latu (2005) found that Pasifika students switch between languages in an effort to understand symbols such as $>$ and $<$. For example, the challenges students face when their language repertoires do not align with the language of instruction are most extreme when the students are simultaneously learning that language in a wider context than just the classroom. In such contexts, there is a need to produce meaningful theories on the learning of mathematics, as well as to develop curricular elements,

Using Children's Literature

When teaching mathematics and statistics teachers are encouraged to use real-life and child-relevant contexts (Perger, 2010). Linking mathematics to children's literature can provide that context, giving links between the world around us and the highly structured learning of mathematics. There are many examples of children's literature suitable for all ages. For example, the story *Fifty-Five Feathers* (Brown, 2004) can be used as motivation for a statistical investigation. This could involve a statistical investigation to find out what birds visit a particular area. Children would have to plan to conduct a survey, what data to collect, how many days to collect the data, where to collect the data, how often to collect. After collecting data and analysing these data, children could use them to answer the initial question. If there is strong interest in the topic, children could contact their local bird experts to find out how their results compare to a wider area of past survey.

Using Technology to motivate students

Hoffert (2008) argues that one way to facilitate the arithmetic skills that English Language Learners lack is to allow them to use calculators. Using calculators can give students confidence in their arithmetic skills so they can focus on the higher-level mathematics necessary to do well in the often-challenging topics. For example, graphing calculators can provide students with a tool for undertaking more complicated and interesting mathematics. For a lesson in statistics once students have collected data and compared the height versus shoe size, a line of best fit could allow students to predict how tall a person would be if he wears a size 37 shoe size.

Another method used in research to help students confront misconceptions is using computer-assisted learning environments. Lesser, Wagler, and Salazar (2016) investigated how a purposeful sample of six (Spanish-speaking) ELLs experienced a bilingual coin-flipping simulation applet (NLVM, 2015) and how students might use such resource to confront content misconceptions and language misunderstandings related to probability concepts covered in college introductory statistics courses.

The students were asked whether they thought using applets like the one they experienced would be helpful to them or other students. All interviewees reported that these applets are beneficial, but reasons varied. For example, P5 reflected:

P5: Well as far as if you are having problems with the wording of certain terms, it would be easier if you have the ability to change language, cause maybe you learned in Spanish like I did, and then can you switch it to English, it helps.

Implications for Practice and Research

This paper shows that dealing with multiple languages in multilingual classrooms is challenging for both students and teachers. Teachers need to be familiar with a range of strategies such as collaborative learning and use of hands-on activities to bridge the language barriers otherwise this situation limits the ability for a learner to answer in anything but English.

Although language may present a barrier to participating in statistics, it also provides a solution. The key to success is ownership of the language and concepts. Students need to be made aware of differences between ordinary English and mathematical/statistical English. Students also need to be given the opportunity to make connections between statistics terms and their own language and experiences, and to use their language through discussion.

Teachers need to realize that often students do not answer a test question because they don't recognize multiple terminology a synonymous term or phrase. Burrill (2008) gave the examples statistics, examples could include "median" versus "second quartile" or "50th percentile," "line of fit" versus "least-squares line" or "regression line," and "z-score" versus "standard score" or "standardized score." When assessing students, teachers should be intentional and explicit about when they are testing for recognition of alternative terminology in addition to the underlying concepts. If not, teachers could be prepared to supply equivalent phrases upon request.

Contextualizing instruction can reinforce meaningful engagement in authentic learning activities, as recommended by a number of authors. However, learning for English Language learning needs to take place in a context that is meaningful to the student (Fischer & Perez, 2008; Gibbons, 1998; Goldenberg, 2008; Winsor, 2007). As mentioned earlier, participants can encounter difficulty with understanding contexts and completing problems when they did not understand the context. For example, ski context created challenges for participants in Lesser and Winsor (2009) study. Even trivial acts such as providing students a picture of what the question is asking can help ELL students understand the concept and complete the problems (Fischer & Perez, 2008). Teacher education institutions will be interested in the findings of this paper. Understanding challenges and some of the opportunity's teachers face in the classroom when teaching learners with a range of languages and language proficiencies, will enable teacher educators to better equip student teachers and teachers to work in multi-lingual and multi-cultural classrooms. Teacher education programmes should integrate multilingual education philosophies, theories, and methodologies in the initial and continuous professional development. Such initiatives will help teachers become familiar with language acquisition as well as teaching theories and teaching techniques.

Increasingly educators around the world are faced with multilingual classrooms as global mobility of populations increases. As might be expected in a maturing field, considerable work needs to be done to map out the scope and develop a coherent understanding of the theoretical diversity brought to work in this area. With increased understanding of ELLs' challenges in statistics, it will be useful to analyze the similarities and differences between effective interventions for ELLs and effective interventions for native English speakers. A particular intervention that would be interesting to explore is whether it is better to define terms formally before explaining a concept (as mathematicians typically do) or, as some studies on language acquisition suggest, informally exploring concepts and then providing students formal language for the concepts being studied (Garrison & Mora, 2003; Murrey, 2008). This strategy can set up the environment of shared learning and ownership in the classroom.

A more long-term direction for research in the arena of ELLs learning statistics would be to apply the situated sociocultural approach of Moschkovich (2002) in mathematics education to statistics education. It would be interesting to observe groups of ELLs at work on statistics problems. What norms do the groups have? How is language used in learning statistics? Do they use English, Spanish, or a combination to talk about statistics problems (Moschkovich, 2007)? What linguistic and cognitive tools do ELLs bring that help them learn statistics? Is a statistics course structured to allow ELLs to interact in ways that promote learning?

It is important to explore tensions that arise in linguistically diverse statistics classrooms: tensions between school and home languages; between academic and informal languages and between language policy and classroom practice; and between a language for learning statistics and a language for getting on in the world. What does linguistic or cultural diversity look like in statistics classroom? How does such diversity influence the teaching or learning of statistics?

Issues raised in this paper stand to contribute to the ongoing discussion and debates about the universality of mathematics/statistics and to influence the teaching and learning of mathematics around the world (Waller, & Flood, 2016). It is hoped that the findings reported in this paper will generate more interest in language challenges

and strategies for English Language Learner in statistics education. Teachers, curriculum developers, and researchers need to continue to work together to find ways to help all students develop statistical literacy.

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