

Optimal Service Strategies in Stochastic Inventory Systems with Reworks for Eligible Customers

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Abstract *this paper presents an inventory system where eligible customers are screened out at the first stage of service. The arrival of demand for fresh items and for rework items follows the Poisson process with parameter λ and β . From fresh items store, items will be provided to the arrival customer within a negligible service time. We assume that a certain portion of arriving customers will get service with rate $\alpha\lambda$ and rest of the arriving customer will be rejected to serve with a rate $(1-\alpha)\lambda$ when inventory level for fresh items reaches to the reorder level s an order takes place which follows exponential distribution with parameter γ . The defective items will dysfunction before expire date, a service will be provided once it returns to the service center with parameter μ . If the store of rework items is full then the next case will be served at home as early as possible. We considered two stores in the system one for fresh items and another for returned items. When inventory level is zero then arrival customer will be lost forever. A suitable mathematical model is developed and the solution of the developed model using Markov process with Rate-matrix is derived. Also the system characteristics are numerically illustrated. The validation of the result in this model was coded in Mathematica 11*

Keywords: *Inventory, Stochastic model, Re-order level, Replenishment, Reworks, Poisson process, exponential distribution.*

1 Introduction

Managing the return items for warranty claimed is one of the key basic leadership challenges in the client driven business world. By merchandise exchange, we comprehend an agreement between the maker and forward positions in the store network (retailers, providers, clients), concerning the system of tolerating back items in the wake of having sold them, either utilized or in an on a par with the new state. Clients restore their arranged things since the things don't satisfy their request. Customer returns of as-good-as-new products have increased dramatically in recent years. Growth in mail-order and transactions over the Internet has increased the volume of product returns as customers are unable to see and touch the items they decide to buy, so they are more likely to return them. Most often the customers discovered that the product they had bought did not have the functionality they expected.

Most of the classical inventory models assume that all items manufactured are of perfect quality. However, in real-life production systems, due to various controllable and/or uncontrollable factors, the generation of defective items during a production run seems to be inevitable, and they should be reworked. In a single stage production system, a certain number of defective items results due to

various reason including poor production quality and material defects and subsequently a portion of them may be scrapped as well. Depending on the portion of defectives, if a number of defective items raise then the optimal batch size varies depending on several cost factors such as setup cost, processing cost, and inventory holding cost. So the production system may have a repair or rework facility at which the defective items will be reworked and/or corrected to finished products. In a production system where there is no repair or rework facility, defective items go to scrap. These defective items are wasted as scraps at each stage in every production cycle, and as a result, many industries having no recycling or reworking facility lose a big share of profit margin.

2 literature review

We center our investigation around stochastic stock management and on those papers, in our view, are basic and build up the structure for future work in one of the headings. In an age system, an explicit piece of insufficient things results in view of various reason, including poor creation quality and material distortions, and thusly a touch of those broken things may be dismissed as well. In the occasion that proportion of flawed things raise, the perfect gathering gauge changes depending upon a couple of cost factors, for instance, setup cost, dealing with cost and stock holding cost finally increase. Customer impatience is a very important phenomenon in Queuing models. Queuing models in which customers may balk, or abandon the system before receiving service arise in many diverse application domains, where customers wait for service for a limited time only and leave the system if service has not begin within that time. In real-time telecommunication systems, the subscribers give up due to impatience before the requested connection is established. Queuing models in which customers leave without getting service either due to mass exodus or due to customers' impatience have been studied in the literature.

Numerous researchers have done them work in this field. **Ammar (2014a)** obtained the transient solution of a two-heterogeneous servers queue with impatient behaviour subject to balking and renegeing.

Armony and Ward (2010) the authors formulated an optimization problem for a call center with heterogeneous agent pools to minimize expected customer is waiting time. They proposed a threshold routing policy which is asymptotically optimal than the common routing policy used in call centers. Recently the authors **Ibrahima et al. (2016)** developed stochastic models and carried out a large-scale databased investigation of service times in a call center with many heterogeneous agents and multiple call types. **Dharmaraja (2000)** with catastrophes, server failures, and repairs and obtained an exact time-dependent solution, however, no work has been found in the literature which studies Queuing systems with heterogeneous service, system disaster and server repair taking together the effect of customers' impatience. Based on this observation, we have investigated the transient system size probabilities for the twoheterogeneous server Queuing system subject to system disaster, server failures, repairs, and customers' impatience by defining a suitable probability generating function, continued fractions (CFs) and identities related to CFs and confluent hypergeometric function. **Dharmaraja S and Kumar R (2015)** studied the Markovian queuing system with heterogeneous servers and catastrophes. The customers arrive according to a Poisson process and the service times follow an exponential distribution. There are a finite number of c servers with different services rates, and each arriving customer requires exactly one server for its service. The queue discipline is FCFS, and the customers select the servers on the fastest server first(FSF) basis. A recent development may be found in the work of **Huel-Hsin Chang et al. (2010)** where they studied the optimal inventory replenishment policy as well as on the long-run production inventory costs. A little attention was paid to the area of imperfect quality EPQ model with backlogging, rework and machine breakdown taking place in stockpiling time. **Ammar (2015)** analyzed a single server queue with impatient customers and multiple vacations, where customers became impatient only due to an absentee of servers upon arrival. **Ammar (2014b)** expands the model described in The motivation of this model also has an application to the dynamic routing problem in computer systems or communication networks. Messages will arrive at the buffer that has to be routed over one of several communication lines. The communication lines are heterogeneous servers which send messages of possibly different speeds. System disaster may occur in the form of virus infections which causes the server down and instantaneously all the messages to be lost. A repair process starts immediately, and the newly arriving messages are queued in the buffer. If the time taken to transmit the message is too long, the sender may give up and leave the system. Our goal is to minimize the overall delay of the message. In **Chung (2011)** developed a supply chain management model and presents a solution procedure to find the optimal production quantity with the rework process. **Chiu, Y.S.P., et al. [13]** developed Mathematical modeling for determining the replenishment policy for a EMQ model with rework and multiple shipments. **Brojeswar Pal et al. (2012)** developed a multi-echelon supply chain model for multiple markets with different selling seasons, and the manufacturer produces a random proportion of defective items, which are reworked after regular production

and are sold in a lot to another market just after completion of rework. **Sudhesh (2010)** derived the transient state probability distribution of the system and obtained an explicit expression using a continued fraction and generating function. **Krisnamoorthi et al. (2013)** developed a single stage production process where defective items produced are reworked and two models of rework processes are considered, an EPQ without shortages, and with shortages, **C.K.Sivashankari, S.Panayappan(2014)** proposed a Production inventory model where they consider reworking of imperfect production, scrap, and shortages. **Kumar (2007)** studied a single server Queuing system with catastrophes, server failures, and repairs. They analyzed the transient behaviour of the model and perform the availability and reliability analysis of the system. **Yechiali (2007)** investigated the single server, homogeneous multi-server and infinite server Queuing systems with system disaster and customer impatience and obtained the stationary probabilities of the system. For the single server model of **Yechiali (2007)**. **Yanyi Xu et al.**, consider a system in which an order is placed every T periods to bring the inventory position up to the base stock S . they accept demand until the inventory position reaches a sales rejection threshold M to find the optimal values of S and M that minimize the long-run average cost per period. **S. K. Goyal and B. C. Giri [10]** presented an inventory model with exponential declining demand and constant deterioration is considered. The time-varying holding cost is a linear function of time. Shortages are not allowed. The items (like food grains, fashion apparels, and electronic equipment) have fixed shelf-life which decreases with time during the end of the season. **S. Priyan and R. Uthayakumar** investigated the issue of an economic manufacturing quantity model for defective products involving imperfect production processes and reworked. They considered that the demand is sensitive to promotional efforts/sales teams' initiatives as well as the setup cost can be reduced through further investment. It also assumed that fixed quantity multiple installments of the finished batch are delivered to customers at a fixed interval of time. **Mohammad Ekramol Islam et al.** Gave a model for non-perishable and the second for perishable things. Their model is more fitting where guarantees are accommodated to a settled time length after the deal for new things was made; they can be used to separate characteristics of the stock system for a broad scale production firm. It was expected that the stock level for both new and the returned things are pre-decided. The requests for both new and the returned things take after the Poisson process. **Sanjoi Sharma et al.** Considered the selling price and then establish an EPQ model, in which the demand rate is a function of the selling price. They also consider the production rate as demand dependent, which was more realistic in our general life. The planning horizon is infinite. The time dependent rate of deterioration was taken into consideration and demand rate was price dependent, shortages are allowed and partially backlogged. Our works are the extension of the **Mohammad Ekramol Islam et al. [16]** which presented an inventory system with postponed demands considering renegeing pools and rejecting Buffer's customers.

3 Assumptions and Notations

3.1 Assumptions

- i. Initially, the inventory level for fresh items is S , and for return items are R .
- ii. Arrival rate of demands follows Poisson process with parameter λ for fresh items and δ for return items.
- iii. Lead-time is exponentially distributed with parameter γ for fresh items.
- iv. If the inventory level for returned items is in R , then home service will be provided for return items.
- v. Service will be provided for the return items with exponential parameter μ .
- vi. A portion of arrival customers will serve with rate $\alpha\lambda$ and rest of them will be rejected with rate $(1-\alpha)\lambda$.

3.2 Notations

- i. S → Maximum inventory level for fresh items.
- ii. R → Maximum inventory level for returned items.
- iii. λ → Arrival rate of demand for fresh items.
- iv. δ → Arrival rate of demands for returned items.
- v. γ → Replenishment rate for fresh items.

- vi. μ →Service rate for returned items.
- vii. $I(t)$ →Inventory level at time t for fresh items.
- viii. $x(t)$ → Inventory level at time t for returned items.
- ix. $E = E_1 \times E_2$ →The state space of the two dimensional Markov process.
- x. Where $E_1 = \{0, 1, 2, \dots, S\}$, $E_2 = \{0, 1, 2, \dots, R\}$

4 Model analysis

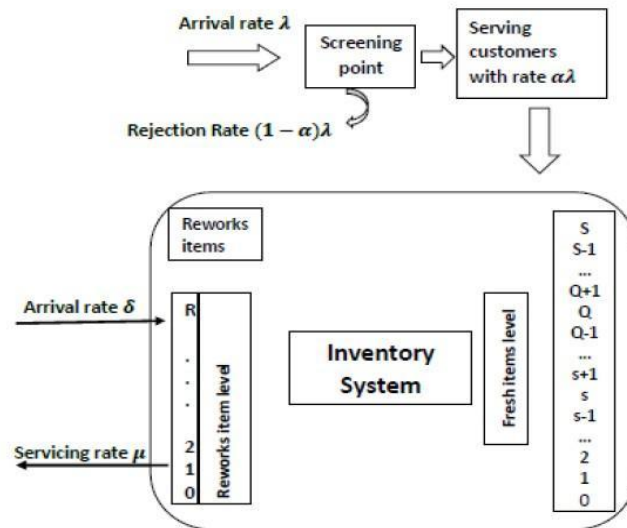


Fig.1: Servicing eligible customer inventory system

In our model, we settled greatest stock dimension for fresh things at S and for return things at R . The entry time between two successive requests are thought to be exponentially dispersed with parameter λ for new things and δ for return things. Each interest is for precisely one unit for everything. At the point when stock dimension diminished to s , a request for replenishment is put. Lead-time is exponentially conveyed with parameter γ . At the point when stock dimension for the arrival things came to at R home service will be accommodated return things.

Now, the infinitesimal generator of the two dimensional Markov process $\{I(t), X(t); t \geq 0\}$ can be defined

$$\tilde{A} = (a(i, j, k, l)); (i, j), (k, l) \in E$$

Hence, we get

$$\tilde{A}(i, j, k, l) = \begin{cases} \alpha\lambda & : i = 1, 2, 3, \dots, S; & k = i - 1, & j = 0, 1, 2, \dots, R, & l = j \\ -(\alpha\lambda + \delta + \mu) & : i = s + 1, s + 2, \dots, S; & k = i, & j = 1, 2, \dots, R - 1, & l = j \\ -(\alpha\lambda + \delta) & : i = s + 1, s + 2, \dots, S; & k = i, & j = 0, & l = j \\ -(\alpha\lambda + \mu) & : i = s + 1, s + 2, \dots, S; & k = i, & j = R, & l = j \\ -(\gamma + \alpha\lambda + \delta) & : i = 1, 2, \dots, s; & k = i, & j = 0, & l = j \\ -\mu & : i = 0; & k = i, & j = 1, 2, \dots, R, & l = j \\ \delta & : i = 0, 1, 2, \dots, S; & k = i, & j = 0, 1, 2, \dots, R - 1, & l = j + 1 \\ \mu & : i = 0, 1, 2, \dots, S; & k = i, & j = 1, 2, \dots, R, & l = j - 1 \\ \gamma & : i = 0, 1, 2, \dots, s; & k = i + Q, & j = 0, 1, 2, \dots, R, & l = j \end{cases}$$

Now, the infinitesimal generator \tilde{A} can be conveniently express as a partition matrix

$\tilde{A} = (A_{ik})$, where A_{ik} is a $(R + 1) \times (R + 1)$ sub-matrix which is given by

$$A_{ik} = \begin{cases} A_1 & \text{if } k = i - 1, i = s + 1, s + 2, \dots, S \\ A_2 & \text{if } k = i, i = s + 1, s + 2, \dots, S \\ A_3 & \text{if } k = i, i = 1, 2, \dots, s \\ A_4 & \text{if } k = i, i = 0 \\ A_5 & \text{if } k = i - 1, i = 1, 2, \dots, s \\ A_6 & \text{if } k = i + Q, i = 0, 1, 2, \dots, s \\ 0 & \text{Otherwise} \end{cases}$$

With

$$A_1 = (a_{ij})_{(R+1) \times (R+1)} = \text{diag } \lambda, \lambda \dots \lambda ; w$$

() here

$$A_2 = (a_{ij})_{(\varphi+1) \times (\varphi+1)} \quad (i, j) \rightarrow (i - 1, j) \text{ for all } i = (s + 1), (s + 2), \dots, S; j = 0, 1, 2, \dots, R$$

$$\begin{aligned} & \cdot (i, j) \cdot (i, j) && \text{is } - (\dots) && \text{for all } i = (s + 1), \dots, S; && j \cdot R \\ & \cdot (i, j) \cdot (i, j) \cdot && \text{is } - (\dots) && \text{for all } i = (s + 1), \dots, S; && j \cdot 1, 2, \dots, R \cdot 1 j \cdot \\ & \cdot (i, j) \cdot (i, j) && \text{is } - (\dots) \text{ is } - && \text{for all } i = (s + 1), \dots, S; && 0 \\ = & \cdot && \text{is } \cdot && \text{for all } i = (s + 1), \dots, S; && \\ & (i, j) && \text{elements} && \text{for all } i = (s + 1), \dots, S; && j \cdot 1, 2, \dots, R j \cdot \\ & \cdot && && \text{are zero} && 0, 1, 2, \dots, R \cdot 1 \\ & \cdot (i, j \cdot 1) && && && \\ & \cdot (i, j) \cdot (i, j \cdot 1) && && && \\ & \cdot && && && \\ & \cdot \text{Other} && && && \end{aligned}$$

$$A_3 = (a_{ij})_{(R+1) \times (R+1)}$$

$$\begin{aligned} & \cdot (i, j) \cdot (i, j) && \text{is } - (\dots) && \text{for all } i = 1, 2, \dots, s; && j \cdot R \\ & \cdot (i, j) \cdot (i, j) \cdot && \text{is } - (\dots) && \text{for all } i = 1, 2, \dots, s; && j \cdot 1, 2, \dots, R \cdot 1 j \\ & \cdot (i, j) \cdot (i, j) && \text{is } - (\dots) \text{ is } - && \text{for all } i = 1, 2, \dots, s; && \cdot 0 \\ = & \cdot && \text{is } \cdot \text{elements} && \text{for all } i = 1, 2, \dots, s; && \\ & (i, j) && && \text{for all } i = 1, 2, \dots, s; && j \cdot 1, 2, \dots, R j \cdot \\ & \cdot && && \text{are zero} && 0, 1, 2, \dots, R \cdot 1 \\ & \cdot (i, j \cdot 1) && && && \\ & \cdot (i, j) \cdot (i, j \cdot 1) && && && \\ & \cdot && && && \\ & \cdot \text{Other} && && && \end{aligned}$$

$$A_4 = (a_{ij})_{(R+1) \times (R+1)}$$

- (i) The mean inventory level for fresh items: $L^1 = \sum_{i=1}^S i \sum_{j=0}^R \pi^{(i,j)}$
- (ii) The mean inventory level for return items $L^2 = \sum_{j=1}^R j \sum_{i=0}^S \pi^{(i,j)}$ b) Re-order rate: Re-order rate for fresh items: $R^0 = \lambda \sum_{j=0}^R \pi^{(s+1,j)}$
- c) Average service rate for return items: $Sr = \mu \sum_{j=1}^R \sum_{i=0}^S \pi^{(i,j)}$
- d) Average customer lost to the system: $CL = \lambda \sum_{j=0}^R \pi^{(0,j)}$
- e) Expected total cost: $ETC = c_1 * L_1 + c_2 * L_2 + c_3 * R_0 + c_4 * Sr + c_5 * CL;$

where c_1 = Holding cost per unit for fresh items, c_2 = Holding cost per unit for return items, c_3 = Replishment cost per order, c_4 = Service Charge for per unit. C_5 = Cost of customer lost for per unit.

7 Numerical illustrations

Putting, $S=5, s=2, R=3, Q=3, \lambda=0.6, \alpha=0.9, \beta=0.02, \mu=0.05, \gamma=0.70, c_1=0.30, c_2=0.40, c_3=0.25, c_4=0.15$ and $c_5=0.50$ Numerical result of the system represent in following figures:

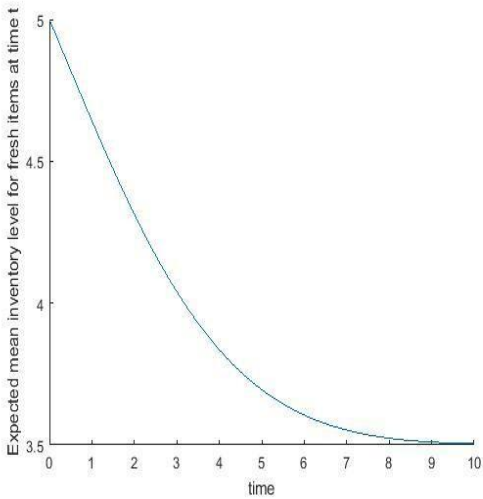


Fig.1: Mean inventory level for fresh items vs time

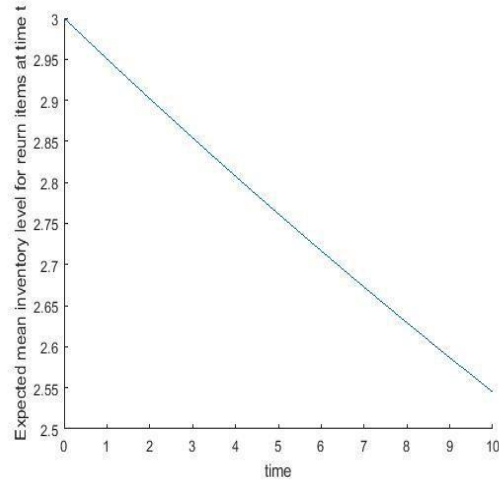


Fig.2: Mean inventory level for return items vs time

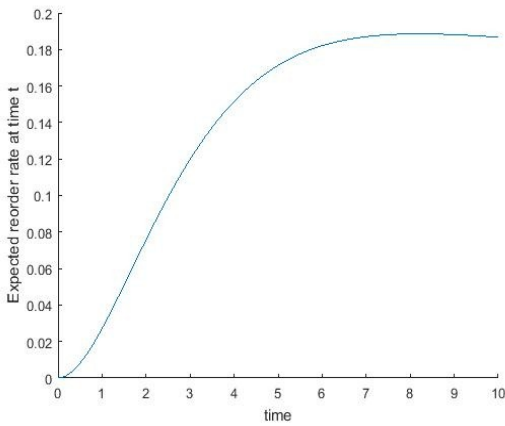


Fig.3: Expected reorder rate vs. time

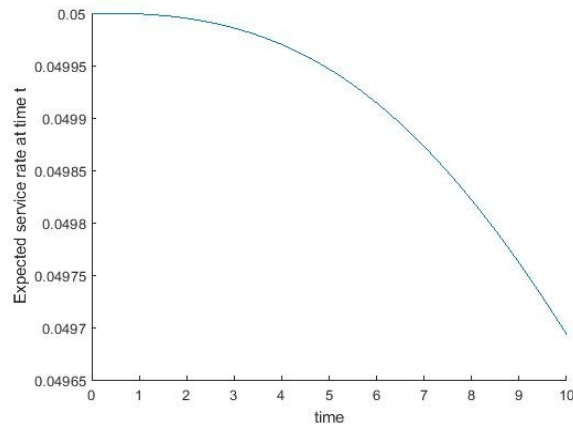


Fig.4: Expected Service rate vs. time

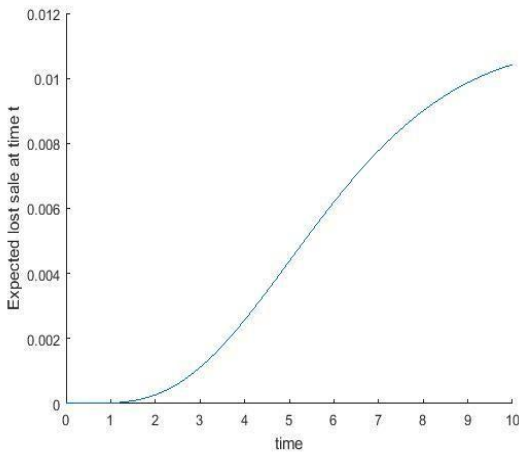


Fig.5: Expected lost sale vs. time

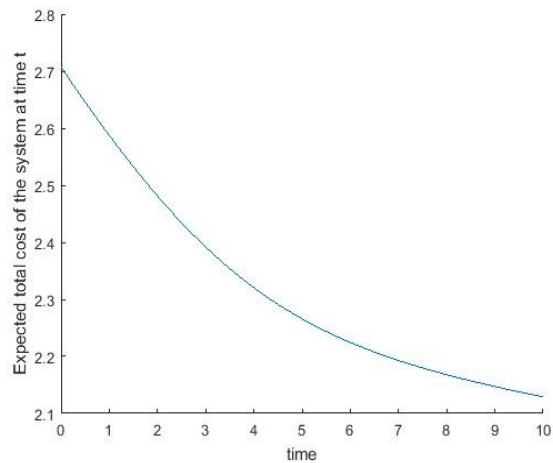


Fig.6: Expected total cost of the system vs. time

8 Sensitivity analyses

All costs related to stock system raise indicate cost. Fig.1 demonstrates that the stock dimension goes downwards adversary some of the time, after that it winds up parallel to x-pivot, which is a result of interest and the impact of renewal. Fig.2 gives the stock dimension for return things, and it diminishes straightly as administration rate is higher than the entry of things. As indicated by Fig.3 reorder recurrence is higher at the beginning stage than that of later on. Fig.4 presents the administration rate which is diminishing gradually with times. Fig.6 demonstrates that that lost deal is similarly low yet is expanding after specific occasions. At

last, we get the framework expected aggregate expense from Fig.6, it diminishing for times and reaches consistent after a prolonged stretch of time.

9 Conclusions

We present transient state attributes of the framework. The decision of qualified customer can reduced stock size and furthermore related cost in a critical propensity. Change process may extend indicate the cost of the system to some degree anyway it incredibly influences checking picture as time goes on. In like manner, it incorporates some fragment of net income. Since holding cost for new things and lost arrangement are progressively fragile, for the headway of affiliation, we should manage these costs cautiously.

References

- i. Ammar SI (2015) Transient analysis of an M/M/1 queue with impatient behavior and multiple vacations. *Appl Math Comput* 260:97–105
- ii. Ammar SI (2014a) Transient analysis of a two-heterogeneous servers queue with impatient behavior. *J Egypt Math Soc* 22(1):90–95
- iii. Ammar SI (2014b) Transient behavior of a two-processor heterogeneous system with catastrophes, server failures and repairs. *Appl Math Model* 38(7–8):2224–2234 Transient analysis of a twoheterogeneous servers queue... 205
- iv. Armony M, Ward AR (2010) Fair dynamic routing in large-scale heterogeneous-server systems. *Oper Res* 58(3):624–637 MR2680568
- v. Brojeswar Pal, ShibSankar Sana and KripasindhuChudhuri, A multi-echelon supply chain model for reworkable items in multiple-markets with supply disruption, *Economic Modeling*, Vol.29, pp.1891-1898.2012
- vi. C.K.Sivashankari, S.Panayappan(2014), Production inventory model with reworking of imperfect production, scrap and shortages, *International Journal of Management Science and Engineering Management*, Vol.9(1), pp.9-20(Taylor's Francies).
- vii. Chiu, Y.S.P., Liu, S.C., Chiu, C.L., Chang, H.M. Mathematical modeling for determining the replenishment policy for a EMQ model with rework and multiple shipments, *Mathematical and Computer Modeling*, 54(9-10), pp.2165-2174, 2011.
- viii. Chung, K.J., the Economic Product Quantity with rework process in supply chain management, *Computers and Mathematics with Application*, 62(6), pp.2547-2550, 2011.
- ix. Dharmaraja S (2000) Transient solution of a two-processor heterogeneous system. *Math Comput Model* 32(10):1117–1123
- x. Dharmaraja S, Kumar R (2015) Transient solution of a Markovian queuing model with heterogeneous servers and catastrophes. *OPSEARCH (Oct–Dec 2015)* 52(4):810–826 .
- xi. Huel-Hsin Chang, Feng-TsungChend, Economic Product Quantity model with backordering, rework and machine failure taking place in stock piling time, *Wseas Transactions on information scienceand applications*, Vol.7Issue4, pp.463-473, 2010.
- xii. Ibrahima R, LEcuyer P, Shenc H (2016) Inter-dependent, heterogeneous, and time-varying servicetime distributions in call centers. *Eur J Oper Res* 250(2):480–492 .
- xiii. Kumar BK, Madheswari SP, Venkatakrishnan KS (2007) Transient solution of an M/M/2 queue with heterogeneous servers subject to catastrophes. *Int J Inform Manag Sci* 18(1):63–80
- xiv. Krishnamoorthi. C and Panayappan, S., An EPQ model for an imperfect production system with rework and shortages, *International Journal of Operation Research*, vol.17(1), pp.104-124, 2013.
- xv. Mohammad Ekramol Islam, K.M Safiqul Islam, M Sharif Uddin, Inventory system with postponed demands considering renegeing pool's and rejecting Buffer's customers, *International Conference on Mechanical, Industrial and Materials Engineering 2013 (ICMIME2013)*, Paper ID: IE-01, 1-3 November, 2013, RUET, Rajshahi, Bangladesh.
- xvi. Mohammad Ekramol Islam, M. Sharif Uddin, Mohammad Ataullah, Stochastic Inventory Models with Reworks, *Yugoslav Journal of Operations Research* 28 (2018), N0. 4, 567-578
- xvii. S. K. Goyal and

- B. C. Giri, Recent trends in modeling of deteriorating inventory, European Journal of Operational Research, vol. 134, no. 1, pp. 1–16, 2001.*
- xviii. S. Priyan and R. Uthayakumar, An EMQ inventory model for defective products involving rework and sales team's initiatives-dependent demand, Journal of Industrial Engineering International (2015) 11:517–529, Springer, DOI 10.1007/s40092-015-0118-6*
- xix. *Sanjoi Sharma, S.R. Singh and Mangey Ram, An EPQ model for deteriorating items with price sensitive demand and shortages, International Journal of Operation Research, vol.23(2), pp.245-255, 2015.*
- xx. *Sudhesh R (2010) Transient analysis of a queue with system disasters and customer impatience. Queueing Syst 66(1):95–105*
- xxi. *Yanyi Xu, Arnab Bisi and Maqbool Dada, A Periodic-Review Base-Stock Inventory System with Sales Rejection, Operations Research, vol. 59, No. 3, P.P. 742–753, 2011, ISSN: 0030-364X (Print), ISSN: 1526-5463 (Online).*
- xxii. *Yechiali U (2007) Queues with system disasters and impatient customers when system is down. Queueing Syst 56(3–4):195–202.*
- xxiii. *Y-S.P. Chiu, C-K. Ting, C-B. Cheng, and F-Y. Lin, Alternative approach for realistic lot-size problem with random scrap rate and backlogging, WSEAS Transactions on Mathematics, Vol. 6 (6), 2007, pp. 736-740.*